

P.R. GOVERNMENT COLLEGE

KAKINADA

An Autonomous Institution & NAAC Accredited with "A" Grade with CGP A 3.17



BOARD OF STUDIES

2019-20

MATHEMATICS



DEPARTMENT OF MATHEMATICS

P.R. GOVERNMENT COLLEGE(A), KAKINADA

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P. R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA, E. G. Dt.

Department of Mathematics

The Board of Studies meeting for Mathematics subject during the academic year 2019-2020 is conducted at the Dept. of Mathematics on 09-04-2019 at 10 a.m. with Dr. V. Anantha Lakshmi, Lect. In-charge in the chair along with the following members.

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Name with Designation and Address

Signature

1. **Chair Person**

Dr. V. Anantha Lakshmi
Lecturer in Mathematics,
P.R. Govt. College (A), Kakinada.

2. **University Nominee**

Dr. B. Sankara Rao,
Assistant Professor & HOD,
College of Science & Technology,
AKNU, Rajahmundry.

3. **Members nominated by Executive council of the College:**

(a) Dr. P.Subhashini,
Lecturer in Mathematics & Principal (FAC),
Government Degree College,
Ramachandrapuram.

(b) Sri. K. Chittibabu,
Lecturer in Mathematics,
Government Degree College,
Ramachandrapuram.

4. **From Alumni:**

(a) Sri. P.S.R.Subrahmanyam,
HOD of Mathematics,
Ideal College of Arts & Science (A),
Kakinada.

5. **Members from the college:**

Faculty members:

- (a) Dr. Sk. Sajana, Lect. in Mathematics
- (b) Ms. V. Hari Priya, G/F in Mathematics
- (c) Smt. G. Subha, G/F in Mathematics
- (d) Smt. P. Maha Lakshmi, G/F in Mathematics

Student Members:

- 1. B. Srinivasu, II M.P.C. (EM)
- 2. B. Vijaya Durga, II M.P.C. (TM)
- 3. S. Krishna Sai, II M.P.Cs. (EM)

(Dr. C. Krishna)

P.R. Government College (Autonomous), Kakinada

Department of Mathematics

Agenda for BOS Meeting in Mathematics

1. Extension of Internal Assessment weightage 40% to III year students also.
2. Revamping of model paper for 5th and 6th semesters.
3. Model question papers, Blue Print and Question Bank.
4. Syllabus, Model Paper and Question Bank for Analytical Skills to all students in IV semester.
5. Additional inputs in to the curriculum.
6. Course outcomes, Programme outcomes and Specific Programme outcomes.
7. Panel of Question Paper Setters and Examiners.
8. Department Action plan for 2019-20.
9. Work load of the department.
10. Any other proposal with the permission of the chair.

BLUE PRINT OF C.B.C.S. MODEL CURRICULUM IN B.Sc. MATHEMATICS

Yr.	Course & Theory / Lab	Paper	Title	Workload Hrs/Week	Credits	Max. Marks		
						Intrnl	Extrnl	Tot
I	Sem I	I	Differential Equations	6 Hrs	5	40	60	100
	Sem II	II	Solid Geometry	6 Hrs	5	40	60	100
II	Sem III	III	Abstract Algebra	6 Hrs	5	40	60	100
	Sem IV	IV	Real Analysis	6 Hrs	5	40	60	100
	All IV Sem. students		Analytical Skills	2 Hrs	2	-	50	50
III	Sem V	V	Ring Theory & Vector Calculus	6 Hrs	5	40	60	100
		VI	Linear Algebra	6 Hrs	5	40	60	100
	Sem VI	VII	Elective (any one)* A. Laplace-Transformations B. Numerical Analysis C. Number Theory D. Graph Theory	6 Hrs	5	40	60	100
		VIII A	1. Integral Transformations 2. Special Functions 3. Project	6 Hrs (for each paper)	5	40	60	100
		VIII B	1. Advanced Numerical Analysis 2. special Functions 3. Project	6 Hrs (for each paper)	5	40	60	100
		VIII C	1. Principles of Mechanics 2. Fluid Mechanics 3. Project	6 Hrs (for each paper)	5	40	60	100
		VIII D	1. Applied Graph Theory 2. Discrete Mathematics 3. Project	6 Hrs (for each paper)	5	40	60	100
	Project		Project work	5 Hrs	5			100

Total number of hours for each paper in the academic year 2019-20:

Paper I & II : 180 Hrs

Paper III & IV : 180 Hrs

Paper V & VI : 150 Hrs

Paper VII : 75 Hrs

Paper VIII (for each paper) : 75 Hrs

P.R. GOVERNMENT COLLEGE (A), KAKINADA
DEPARTMENT OF MATHEMATICS

Objectives of Department

- To impart knowledge on various Mathematical concepts like Differential Equations, Solid Geometry, Group Theory, Real Analysis, Ring Theory and Vector Calculus, Linear Algebra, Numerical Analysis and Special Functions.
- To equip our students with good quality to appear for competitive examinations.
- To make the students to understand the needs of Mathematics in Science and Technology.
- To inculcate research atmosphere among students by assigning projects.

The Department of Mathematics is offering B.Sc. courses involving mathematics (10 courses), B.Sc. Professional (B.Voc.) for undergraduate courses.

B.Sc. MATHEMATICS STREAM COURSES

PROGRAMME OUTCOMES

For every degree program expectations are listed out by the institution under the Program Outcomes. For B.Sc. Mathematics Stream the following are set as Programme Outcomes.

PO1. Knowledge and Understanding of:

1. All concepts in under graduate level.
2. Real life applications of these concepts and relationship between them.

PO2. Intellectual skills – be able to:

1. Think logically and arrange real life situations to mathematical problems.
2. Assimilate knowledge and ideas based on wide reading and through the internet.
3. Transfer of appropriate knowledge and methods from one topic to another within the subject.
4. Understand the evolving state of knowledge in a rapidly developing field.

PO3. Transferable skills:

1. Use of IT (word-processing, use of internet for doing project).
2. Ability to work as part of a team.
3. Ability to use library resources/Equipment.
4. Time management.

PO4. Problem analysis:

1. Conversion of real life problem to Mathematical problem to get solution.
2. Conduct investigations of complex problems: Use research-based knowledge.

PO5. Ethics:

1. Apply ethical principles, commit environment and responsibilities among students.

PO6. Individual and team work:

1. Function effectively as an individual and as a member in diverse teams, and in multidisciplinary settings.

PO7. Communication:

1. Communicate effectively on complex group activities and with society at large. Speak, read, write and listen clearly in person and through electronic media in English.

PO8. Critical Thinking:

1. Take informed actions after identifying the assumptions that frame our thinking and actions, checking out the degree to which these assumptions are accurate and valid, and looking at our ideas and decisions (intellectual, organizational, and personal) from different perspectives.

PO9. Effective Citizenship:

1. Demonstrate empathetic social concern and equity centred national development, and the ability to act with an informed awareness of issues and participate in civic life through volunteering.

PO10. Life-long learning:

1. Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAMME SPECIFIC OUTCOMES OF MATHEMATICS

PSO1. Identify and understand all the Mathematical concepts in under graduate level.

PSO2. To understand the applications of these concepts in real life.

PSO3. Identify and understand the applications of these concepts in different areas like Physics, Chemistry, Computer Science, etc.

PSO4. Identify the relationship between different concepts like Group theory, Ring theory and Linear Algebra.

Courses (Papers) offered under B.Sc. Mathematics Stream

S. No.	Sem. No.	Domain Specific course/Clusters	Title
1	I	General Core	Differential Equations
2	II	General Core	Solid Geometry
3	III	General Core	Abstract Algebra
4	IV	General Core	Real Analysis
5	V	General Core	Ring Theory & Vector Calculus
6		General Core	Linear Algebra
7	VI	Elective B	Numerical Analysis
8		Cluster Elective B1	Advanced Numerical Analysis
		Cluster Elective B2	Special Functions

Course Outcomes

I Semester – Differential Equations

- CO1. To understand the concept and apply appropriate methods for solving equations.
- CO2. To get awareness and skills to apply the differential equations in various fields.

II Semester – Solid Geometry

- CO1. To understand the applications of three dimensional geometry.
- CO2. To express the problems geometrically and then to get the solution.

III Semester – Abstract Algebra

- CO1. To analyse the abstract concepts and prove the theorems.
- CO2. To understand the applications of group theory in various fields.

IV Semester – Real Analysis

- CO1. To get clear idea about the real numbers and real valued functions.
- CO2. To get clear idea about the real numbers and real valued functions.
- CO3. To obtain the skills of analyzing the concepts and applying appropriate methods for testing converges of a sequence or series.
- CO4. To test the continuity and differentiability and Riemann integration of a function.
- CO5. To know the Geometrical interpolation of mean value theorems.

CO6. With the knowledge of sequence and series, to solve the series completion and finding odd man out in analytical skills.

V Semester – Ring Theory & Vector Calculus

CO1. To understand the ring theory concepts with the help of knowledge in group theory and to prove the theorems.

CO2. To understand the applications of ring theory in various fields.

CO3. To get knowledge of vector differentiation, differential operators, line surface and volume integrals.

CO4. To obtain the skills to convert one type of integral to other types and to solve the problems.

CO5. To solve the problems on applications of gradient, diverge and curl.

V Semester – Linear Algebra

CO1. To understand the concepts of linear algebra.

CO2. To improve the skills of analyzing the concepts, to proving the theorems and solving problems.

CO3. To apply the concepts in different fields.

VI Semester – Numerical Analysis & Advanced Numerical Analysis

CO1. To understand the concepts and obtain the interpolation formulae.

CO2. To understand the different concepts in Numerical Analysis, i.e., finding the solution of equations and deriving the interpolation formula, curve fitting, solving system of equations and solving ordinary differential equations.

CO3. To get skills of applying these concepts in different areas.

VI Semester – Special Functions

CO1. To know the concepts of β, γ functions, the relation between theorems and solving the problems.

CO2. To obtain Hermite polynomial, Laguerre polynomial, Legendre's polynomial and Bessel's functions by solving the concerned differential equations and to solve problems on these functions.

Internal Assessment

Paper I, II, III, IV, V, VI, VII & VIII:

Weightage for Internal Assessment is 40 marks.

For Mid Semester Examinations - 20 marks

For Continuous Assessment - 20 marks

Two Mid Semester Examinations will be conducted for 40 marks in the following

Question Paper pattern:

Objective Questions (2 mark) : 08 - $8 \times 2 = 16$ marks

Short answer question (6 marks) : 04 - $4 \times 6 = 24$ marks (internal choice)

40 marks

The average of two mid examination marks are to be taken for 20 marks.

For continuous assessment – 20 marks distributed in the following way:

Assignment - 10 marks

Seminar - 5 marks

Viva voce exam - 5 marks

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I B.Sc. MATHEMATICS-SEMESTER I (w.e.f. 2017-2018)
COURSE: DIFFERENTIAL EQUATIONS

Total Hrs. of Teaching-Learning: 90 @ 6 hr/Week

Total credits: 05

OBJECTIVES:

- To classify differential equations by order, linearity and homogeneity.
- To compute solutions to various differential equations by using analytical techniques.
- To identify the appropriate method for solving the given differential equation.
- To get awareness about the applications.

Unit I: Differential Equations of First Order and First Degree (18 hours)
Exact differential equations, integrating factors, linear Differential equations, Differential equations reducible to linear form, Change of variables.

Unit II: Orthogonal Trajectories, Differential Equations of the First Order But Not of the First Degree (18 hours)
Orthogonal Trajectories, Equations solvable for p, Equations solvable for y, Equations solvable for x, Equations that do not contain x (or y), Clairaut's equation.

Unit III: Higher Order Linear Differential Equations (with constant coefficients) - I (18 hours)
Solution of homogeneous linear differential equations of order n with constant coefficients. Solution of non-homogeneous linear differential equations with constant coefficients $f(D)y = Q(x)$ by means of polynomial operators when $Q(x) = be^{ax}$, $Q(x) = b \sin ax$ or $b \cos ax$.

Unit IV: Higher Order Linear Differential Equations (With Constant Coefficients) -- II (18 hours)
Solution of the non-homogeneous linear differential equations with constant coefficients $f(D)y = Q(x)$ by means of polynomial operators when $Q(x) = bx^k$, $Q(x) = e^{ax}V$, $Q(x) = xV$ and $Q(x) = x^mV$.

Unit V: Higher Order Linear Differential Equations: (With Non Constant Coefficients) (18 hours)
Method of variation of parameters, Linear differential equations with non-constant coefficients, The Cauchy-Euler equation.

Additional Inputs:

1. Simultaneous differential equations
 2. Applications of 1st order and 1st degree differential equations.
- (No question to be set from this part)

Prescribed Text Books:

1. Scope as in "Differential Equations and their applications by Zafar Ahsan, published by prentice-Hall of India Pvt. Ltd. New Delhi-Second edition.

Reference Books:

1. A text book of Mathematics, Volume-I published by S. Chand & Company.
2. Differential Equations by Santhi Narayana, S. Chand & Company.

**BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-I**

Unit	TOPIC	V.S.A.Q 1M	S.A.Q 5M	E.Q 8M	Marks allotted
I	Differential Equations of 1 st order and 1 st degree	1	1	2	22
II	Orthogonal Trajectories, Differential Equations of 1 st order but not of 1 st degree	1	1	2	22
III	Higher Order Linear Differential Equations (with constant coefficients) – I	1	1	1	14
IV	Higher Order Linear Differential Equations (with constant coefficients) – II	1	1	2	22
V	Higher Order Linear Differential Equations (with non constant coefficients)	1	1	1	14
TOTAL		5	5	8	94

- V.S.A.Q. = Very short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q. = Essay questions (8 marks)

Very short answer questions : 5 X 1 = 05

Short answer questions : 3 X 5 = 15

Essay questions : 5 X 8 = 40

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Total Marks = 60
.....

P.R. Government College (Autonomous), Kakinada
I year B.Sc., Degree Examinations – I Semester
Mathematics Course: Differential Equations
Paper I (Model paper w.c.f. 2017-2018)

Time: 2Hrs 30 min

Max. Marks: 60

PART-I

Answer ALL the questions. Each question carries 1 mark.

5 X 1 = 5 M

1. Write the condition for a differential equation of first order to be an exact differential equation.
2. Solve $(p - x)(p - y^2) = 0$.
3. Find y_c of the differential equation $(D^2 + 4D + 4)y = 3xe^{-2x}$.
4. Find the particular integral of $D^2y = x^2$.
5. In a D.E. $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$, if $1 + P + Q = 0$ then what is a part of complementary function.

PART-II

Answer any THREE questions, each question carries 5 marks.

3 X 5 = 15 M

6. Solve $(e^y + 1)\cos x dx + e^y \sin x dy = 0$.
7. Solve $(py + x)(px - y) = 2p$.
8. Solve $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = \sin 2x$.
9. Solve $(D^2 - 2D + 1)y = x^2 e^{3x}$.
10. Solve $(D^2 - 2D)y = e^x \sin x$, by the method of variation of parameters.

PART-III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks.

5 X 8 = 40 M

SECTION-A

11. Solve $(y + \frac{y^3}{3} + \frac{x^2}{2})dx + \frac{1}{4}(x + xy^2)dy = 0$.
12. Solve $(1 + y^2)dx = (\tan^{-1}y - x)dy$.
13. Solve $y^2 \log y = xpy + p^2$.
14. Find the orthogonal trajectories of the family of curves $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, where 'a' is a parameter.

SECTION-B

15. Solve $(D^2 - 4D + 3)y = \sin 3x \cdot \cos 2x$.
16. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x} \sin 2x$.
17. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^x \sin x$.
18. Solve $(x^2 D^2 + xD - 1)y = x^2 e^x$.

PR GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank
PAPER-I: DIFFERENTIAL EQUATIONS

Very Short Answer Questions

Unit-I

1. Write the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact.
2. Check whether the differential equation $(e^y + 1)\cos x dx + e^y \sin x dy = 0$ is exact or not.
3. Check whether the differential equation $(x^2 - ay)dx = (ax - y^2)dy$ is exact or not.
4. $\frac{xdy + ydx}{xy} = \underline{\hspace{2cm}}$.
5. Solve $xdy - ydx = xy^2 dx$ by inspection method.
6. Solve $ydx - xdy + \log x dx = 0$ by inspection method.
7. Solve $xdy - (y - x)dx = 0$ by inspection.
8. If $Mdx + Ndy = 0$ is a homogenous equation such that $Mx + Ny \neq 0$ then the integrating factor of the differential equation is $\underline{\hspace{2cm}}$.
9. Find the integrating factor of $x^2 y dx - (x^3 + y^3) dy = 0$
10. Find the integrating factor of $y(xy + 2x^2 y^2) dx + x(xy - x^2 y^2) dy = 0$.
11. Find the integrating factor of $y(1 + xy) dy + x(1 - xy) dy = 0$.
12. If $Mdx + Ndy = 0$ is not exact and $\frac{1}{M} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) = -\frac{3}{y}$ then an integrating factor of $Mdx + Ndy = 0$ is $\underline{\hspace{2cm}}$.
13. Find the integrating factor of $(x^2 + 1) \frac{dy}{dx} + 4xy = \frac{1}{x^2 + 1}$.
14. Find the integrating factor of $(1 + x) \frac{dy}{dx} - xy = 1 - x$.
15. Find the integrating factor of $(1 + y^2) dx = (\tan^{-1} y - x) dy$.
16. Reduce the given Bernoulli's differential equation $\frac{dy}{dx} + \frac{y}{x} = y^2 x \sin x$ into the linear differential equation.

Unit-II

17. Find the orthogonal trajectory of the family of curves $y = cx$ where c is the parameter.
18. Define self-orthogonal family of curves.
19. Find the orthogonal trajectory of the family of curves $r = a\theta$ where a is the parameter.
20. Solve $p^2 - 7p + 10 = 0$.
21. Solve $p^2 x^2 = y^2$.
22. Solve $(y - xp)(p - 1) = p$.
23. Solve $\sin(y - px) = p$.
24. Solve $p = \log(px - y)$.

Unit-III

25. Solve $\frac{d^3 y}{dx^3} - 3 \frac{dy}{dx} + 2y = 0$.
26. Solve $(D^4 + 8D^2 + 16)y = 0$.
27. Find the particular value of $\frac{1}{D} x^2$
28. Find the particular value of $\frac{1}{D^2 - 5D + 6} e^{2x}$
29. Find y_c of the differential equation $(D^2 + 4D + 4)y = 3xe^{-2x}$.

30. Find y_p of $(D^2 + 3D)y = 6$.

31. find the particular integral of $\frac{1}{D^2+9} \sin 3x$.

32. Find the particular integral of $\frac{1}{D^2+4} \cos 2x$.

Unit-IV

33. Find the particular intergral of $D^2y = x^2$.

34. Find the particular value of $\frac{1}{D+1} x$.

35. Find the particular value of $\frac{1}{D^2+D+1} x^3$.

36. Find the particular integral of $(D + 2)y = xe^{-2x}$.

37. Find the particular integral of $(D^2 + 1)y = \sin 2x$.

Unit-V

38. In the method of variation of parameters the particular integral of $\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = R$ is written as $y_p = Au + Bv$ where $A = \underline{\hspace{2cm}}$, $B = \underline{\hspace{2cm}}$.

39. Find the part of C.F. of $(x - 1) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 1$.

40. In a D.E. $\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = R$, if $1+P+Q=0$ then what is the part of complementary function.

41. Write the general form of Cauchy-Euler Equation.

Short Answer Questions

Unit-I

1. Solve $x \frac{dy}{dx} + 2y - x^2 \log x = 0$

2. Obtain the equation of the curve satisfying the differential equation

$$(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0.$$

3. Solve $\frac{dy}{dx} + 2xy = e^{-x^2}$.

4. Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$.

5. Solve $(1 + y^2)dx = (\tan^{-1}y - x)dy$.

6. Solve $x \frac{dy}{dx} + y \log y = xye^x$

7. Solve $(1 + e^{x/y})dx + e^{x/y} \left(1 - \frac{x}{y}\right)dy = 0$.

8. Solve $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0$

9. Solve $(x^2 + y^2 + 2x)dx + 2y dy = 0$.

10. Solve $(1 + xy)y dx + (1 - xy)x dy = 0$.

11. Solve $(x^2 - 2xy - y^2)dx - (x + y)^2 dy = 0$.

12. Solve $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$.

13. Define an Integrating factor and solve $xdy - ydx = xy^2 dx$.

14. Solve $xdy = [y + x \cos^2(y/x)]dx$

15. Solve $ydx - xdy + (1 - x^2)dx + x^2 \sin y dy = 0$

Unit-II

- Find the Orthogonal trajectories of the family of coaxial circles $x^2 + y^2 + 2fy - 1 = 0$, f being the parameter.
- Find the orthogonal trajectories of the family of $r = a(1 - \cos \theta)$ where 'a' is the parameter.
- Solve $y = 2px + x^2p^4$.
- Define Clairaut's equation and solve $(px - y)(py + x) = 2p$.

Unit-III

- Solve $(D^2 - 3D + 2)y = \text{Cosh}x$.
- Solve $(D^2 + 9)y = \text{Cos } 3x$.
- Solve $(D^2 - 5D + 6)y = e^{4x}$.
- Solve $(D^2 - D - 2)y = \text{Sin}2x$.
- Solve $(D^2 + 4)y = \text{Sin}2x$.

Unit-IV

- Solve $(D^2 - 2D + 1)y = x^2e^{3x}$.
- Solve $(D^2 + 4)y = x\text{Sin}x$.
- Solve $(D^3 + 2D^2 + D)y = e^{2x} + x$.
- Solve $(D^2 - 4D + 4)y = x^3$
- Solve $(D^4 + D^2)y = 4\text{Sin}x - 2\text{Cos}x$

Unit-V

- Solve $(D^2 + 1)y = \text{Sec}x$ by method of variation of parameters.
- Solve $(D^2 - 3D + 2)y = \text{Cos } e^{-x}$ by the method of variation of parameters.
- Solve $(x^4D^3 + 2x^3D^2 - x^2D + x)y = 1$
- Solve $(x^3D^3 + 2x^2D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$.

Essay Answer Questions

Unit-I

- Solve $x \cos x \frac{dy}{dx} + (x \sin x + \cos x)y = 1$
- Solve $\text{Cos}^2x \frac{dy}{dx} + y = \tan x$.
- Solve $\frac{dy}{dx} + 2xy = e^{-x^2}$.
- Define Bernoulli's equation of first order and solve $x \frac{dy}{dx} + y = y^2 \log x$.
- Solve $\frac{dy}{dx}(x^2y^3 + xy) = 1$
- Solve $ydx - xdy + (1 - x^2)dx + x^2 \sin y dy = 0$
- Solve $x^2y dx - (x^3 + y^3)dy = 0$.
- Solve $y^2dx + (x^2 - xy - y^2)dy = 0$.
- Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.
- Solve $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$.
- Solve $(x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)xdy = 0$
- Solve $(xy \sin xy + \cos xy)y dx + (xy \sin xy - \cos xy)x dy = 0$.
- Solve $(x^3y^3 + x^2y^2 + xy + 1)y dx + (x^3y^3 - x^2y^2 - xy + 1)x dy = 0$.
- Solve $(1 + xy)x dy + (1 - xy)ydx = 0$.

15. Solve $(y + \frac{y^3}{3} + \frac{x^2}{2}) dx + \frac{1}{4}(x + xy^2) dy = 0$.
16. Solve $(x^3 - 2y^2) dx + 2xy dy = 0$.
17. Solve $(xy^3 + y) dx + 2(x^2y^2 + x + y^4) dy = 0$
18. Solve $(y^4 + 2y) dx + (xy^3 + 2y^4 - 4x) dy = 0$.

Unit-II

19. Show that the family of confocal conics $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$ is self-orthogonal, where λ is the Parameter.
20. Find the Orthogonal trajectories of the family of curves in polar coordinates $r = \frac{2a}{1 + \cos \theta}$ Where 'a' is the parameter?
21. Solve $p^2 + 2py \cot x = y^2$
22. Solve $y = 2xp + y^2 p^3$
23. Solve $y + px = p^2 x^4$
24. Solve $y^2 \log y = xpy + p^2$

Unit-III

25. Solve $(D^2 - 4D + 3)y = \sin 3x \cos 2x$.
26. Solve $(D^2 + a^2)y = \sec ax$
27. Solve $(D^2 + a^2)y = \tan ax$
28. Solve $(D^2 + 4)y = e^x + \sin 2x + \cos 2x$.
29. Solve $(D^2 + 4)y = \sin 2x$

Unit-IV

30. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x} \sin 2x$
31. Solve $(D^2 + 4)y = x^2 e^{3x} + e^x \cos 2x$
32. Solve $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$
33. Solve $(D^4 + 2D^2 + 1)y = x^2 \cos x$
34. Solve $(D^2 - 2D + 1)y = x e^x \sin x$

Unit-V

35. Solve $(x^3 D^3 + 2x^2 D^2 + 2)y = 10 \left(x + \frac{1}{x} \right)$.
1. Solve $(x^2 D^3 + 3x D^2 + D)y = x^2 \log x$.
2. Solve $(x^2 D^2 + 3x D + 1)y = \frac{1}{(1-x)^2}$
3. Solve $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - (1 - \cot x)y = e^x \sin x$
4. Solve $(D^2 + a^2)y = \sec ax$ by method of variation of parameters.
5. Solve $y'' + 4y = 4 \tan 2x$ by the method of variation of parameters.
6. Solve $(D^2 + 1)y = \operatorname{Cosec} x$ by method of variation of parameters.
7. Solve $(D^2 + a^2)y = \tan ax$ by method of variation of parameters.
8. Solve $(D^2 - 2D)y = e^x \sin x$ by method of variation of parameters.

P.R.GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
I B.Sc. – MATHEMATICS - SEMESTER II (w.e.f. 2017-2018)

Course: SOLID GEOMETRY

Total Hrs. of Teaching-Learning: 90 @ 6 h / Week

Total Credits: 05

Objective:

- To get awareness about the three dimensional geometry along with visualization.
- To be able to apply 3-D geometry for the construction.

Unit I: The Plane (18 h)

Equation of plane in terms of its intercepts on the axes, Equation of the plane through the given points, Length of the perpendicular from a given point to a given plane, Bisectors of angles between two planes, Combined equation of two planes, Orthogonal projection on a plane.

Unit II: The Straight Line (18 h)

Equation of a line, Angle between a line and a plane, the condition that a given line may lie in a given plane, the condition that the given lines are coplanar, Number of arbitrary constants in the equations of straight line, sets of conditions which determine a line, The shortest distance between two lines, the length and equations of the line of shortest distance between two straight lines, length of the perpendicular from a given point to a given line.

Unit III: The Sphere (18h)

Equation of the sphere, Plane section of a sphere, Intersection of two spheres, Equation of a circle, Sphere through a given circle, Intersection of a sphere and a line, Tangent lines and tangent planes, Plane of contact, Polar plane, conjugate points, Conjugate planes.

Unit IV: The Sphere and the Cone (18h)

Angle of intersection of two spheres, Condition for two spheres to be orthogonal, Radical plane, Coaxial system of spheres, Simplified form of the equation of two spheres.

Definition of a cone, Vertex, Guiding curve, generators, Equation of the cone with a given vertex and guiding curve, Equation of cone with vertex at origin is homogeneous, Condition that the general equation of the second degree should represent a cone.

Unit V: The Cone (18 h)

Enveloping cone of a sphere, Right Circular Cone, Conditions that a cone may have three mutually perpendicular generators, Intersection of a line and quadric cone, Tangent lines and tangent plane at a point, Condition that a plane may touch a cone, Reciprocal cones, Intersection of two cones with a common vertex.

Additional Inputs:

1. Intersection of three planes, Triangular prism.
2. The right circular cylinder.

Prescribed Book:

Scope as in "A text book of Mathematics for B.Sc. volume I" by V. Krishna Murthy & others, S.Chand and Company Ltd.

Reference Books:

1. Analytical Solid Geometry by Shanti Narayan and P. K. Mittal, Published by S. Chand & Company Ltd., Seventh Edition.
2. A text book of Analytical Geometry of Three Dimensions by P. K. Jain and Khaleel Ahmed, Wiley Eastern Ltd., 1999.
3. Course on Solid Geometry by N. P. Bali - Golden series publications.

**BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-II**

Unit	TOPIC	V.S.A.Q	S.A.Q	E.Q	Marks allotted to the Unit
I	The Plane	1	1	2	22
II	The Right Line	1	1	2	22
III	The Sphere	1	1	1	14
IV	The Sphere & The Cone	1	1	2	22
V	The Cone	1	1	1	14
TOTAL		5	5	8	94

V. S. A. Q. = Very short answer questions (1 mark)

S. A. Q. = Short answer questions (5 marks)

E.Q. = Essay questions (8 marks)

Very short answer questions : 5 X 1 = 05

Short answer questions : 3 X 5 = 15

Essay questions : 5 X 8 = 40

.....
Total Marks = 60
.....

P.R. Government College (Autonomous), Kakinada
I year B.Sc., Degree Examinations - II Semester
Mathematics Course: Solid Geometry
Paper II (Model Paper w.e.f. 2017 - 2018)

Time: 2Hrs 30 min

Max. Marks: 60

PART-I

Answer **ALL** the questions. Each question carries 1 mark. 5 X 1 = 5 M

1. Find the equation of the plane through the line of intersection of $x - 3y + 2z + 3 = 0$, $3x - y - 2z - 5 = 0$ and the origin.
2. Find the equation of the line passing through $(4, 3, -7)$ and equally inclined to the axes.
3. Find the centre of the sphere $x^2 + y^2 + z^2 - 3x + 5y - 4z - 3 = 0$.
4. Find the polar plane of the point $(0, -1, 1)$ with respect to the sphere $x^2 + y^2 + z^2 - 2x + 4y + 6z - 11 = 0$.
5. Write the reciprocal cone of $9x^2 + 4y^2 - 7z^2 = 0$.

PART -II

Answer any **THREE** questions. Each question carries 5 marks. 3 X 5 = 15 M

6. Find the equation of the plane through the point $(-1, 3, 2)$ and perpendicular to the two planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$.
7. Find the image of the point $A(1, 3, 4)$ in the plane $2x - y + z + 3 = 0$.
8. Find the equation of the sphere through the origin and making intercepts a, b, c with the axes.
9. If r_1 and r_2 are the radii of the orthogonal spheres, then find the radius of the circle of their intersection.
10. Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$, with its vertex at $(1, 1, 1)$.

PART-III

Answer any **FIVE** questions from the following by choosing at least **TWO** from each section. Each question carries 8 marks. 5 X 8 = 40 M

SECTION -A

11. Find the planes bisecting the angles between the planes $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$. Point out which of the planes bisects the acute angle and which bisects the obtuse angle in which the origin lies.
12. Show that the equation $x^2 + 4y^2 + 9z^2 - 12yz - 6zx + 4xy + 5x + 10y - 15z + 6 = 0$ represents a pair of parallel planes and find the distance between them.
13. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$; $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.
14. Find the length and equations of the line of shortest distance between the lines $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

SECTION -B

15. Show that the four points $(-8, 5, 2)$, $(-5, 2, 2)$, $(-7, 6, 6)$, $(-4, 3, 6)$ are concyclic.
16. Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at $(1, -2, 1)$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.
17. Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in a perpendicular lines if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.
18. Find the equation to the right circular cone whose vertex in $P(2, -3, 5)$ axis PQ which makes equal angles with the axis and which passes through $(1, -2, 3)$.

PR GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank
PAPER-II: SOLID GEOMETRY

Very Short Answer Questions
UNIT-I

1. Find the intercepts of the plane $2x - 3y + 4z = 12$ on the coordinate axes.
2. Find the normal form of the equation of the plane $2x - 3y + 4z = 12$.
3. Find the equation of the plane through the line of intersection of $x - 3y + 2z + 3 = 0$, $3x - y - 2z - 5 = 0$ and the origin.
4. Find the distance between the parallel planes $3x - y + 2z + 4 = 0$, $6x - 2y + 4z + 5 = 0$.
5. Find the perpendicular distance from $(2, 3, 1)$ to the plane $2x + 2y - z + 5 = 0$.
6. Find the equation of the plane passing through $(2, 3, 5)$ and whose normal has the Direction ratios $(1, 2, 3)$.
7. Find the equation of the plane passing through $(4, 0, 1)$ and parallel to the plane $4x + 3y - 12z + 6 = 0$.
8. Check whether the points $(1, -1, 3)$, $(3, 3, 3)$ lie on the same side or opposite sides of the plane $5x + 2y - 7z + 9 = 0$.
9. Find the angle between the planes $2x - 3y + 4z + 11 = 0$, $3x - 2y - 3z + 27 = 0$.

UNIT-II

10. Find the equation of the line through (α, β, γ) and parallel to X-axis.
11. Find the equation of the line joining $(-2, 1, 3)$, $(1, 1, 4)$.
12. Find the angle between the lines $\frac{x-2}{1} = \frac{y-4}{0} = \frac{z-5}{-1}$, $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$.
13. Find k so that the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$, $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular.
14. Find equation of the line passing through $(4, 3, -7)$ and equally inclined to the axis.
15. Show that the line $\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z+4}{5}$ is parallel to $3x + 4y + z = 4$.
16. Show that the line $\frac{x+3}{-2} = \frac{y+4}{3} = \frac{z+4}{5}$ is perpendicular to the plane $-2x + 3y + 5z = 0$.

UNIT-III

17. Find the centre of the sphere $x^2 + y^2 + z^2 - 3x + 5y - 4z - 3 = 0$.
18. Find the radius of the sphere $x^2 + y^2 + z^2 - 3x + 5y - 4z - 3 = 0$.
19. Find t , if the radius of the sphere $x^2 + y^2 + z^2 + 6x - 8y - t = 0$ is 6.
20. Find the equation of the sphere of radius 3 concentric with the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 1 = 0$.
21. Define great circle and small circle.
22. Find the length of the tangent line from the point $(3, 1, -1)$ to the spheres $x^2 + y^2 + z^2 - 3x + 5y + 7 = 0$.
23. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ at $(-1, 4, -2)$.
24. Find the equation of the sphere having $(2, 3, -1)$, $(1, -2, -1)$ as ends of the diameter.
25. Find the polar plane of the point $(0, -1, 1)$ w.r.to the sphere $x^2 + y^2 + z^2 - 2x + 4y + 6z - 11 = 0$.

UNIT-IV

26. Define angle of the intersection of two spheres.
27. What is the equation of the radical plane of two non concentric spheres $S=0$, $S'=0$?

28. Define limiting points coaxial system of spheres.
29. Show that the spheres $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$, $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ are orthogonal.
30. Find the radical plane of the two spheres $x^2 + y^2 + z^2 + 3x + y - z + 6 = 0$ and $x^2 + y^2 + z^2 + 6x + 4y + 2z - 8 = 0$.
31. Define a cone.
32. Equation of a cone with vertex at origin and the guiding curve $x^2 + y^2 = 9, z = 3$.
33. Show that $x = -y = -z$ is a generator of the cone $5yz + 8zx + 3xy = 0$.

UNIT-V

34. Write the condition for a quadric cone have three mutually perpendicular generators.
35. Find the semi vertical angle of a right circular cone having $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ as the axis and x-axis is a generator.
36. Write the reciprocal cone of $9x^2 + 4y^2 - 7z^2 = 0$.
37. If a cone has three mutually perpendicular tangent planes then its reciprocal cone has three _____.

Short Answer Questions

UNIT-I

- Find the equation of the plane passing through the points (2, 2, -1), (3, 4, 2), (7, 0, 6).
- Find the equation of the plane through the point (-1, 3, 2) and perpendicular to the planes $x+2y+2z=5$ and $3x+3y+2z=8$.
- Find the equation to the plane through the points (1, 1, 1), (1, -1, 1) and (-7, -3, -5). Show that it is parallel to y- axis.
- Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y - z = -4$ and is parallel to X axis.

UNIT-II

- Find the equation of the line through the point (1, 2, 3) and parallel to X- axis.
- Find the image of the point A(1, 3, 4) in the plane $2x - y + z + 3 = 0$.
- Find the equations of the line through the point (1, 1, 1) and intersecting the lines $2x - y - z - 2 = 0 = x + y + z - 1$; $x - y - z - 3 = 0 = 2x + 4y - z - 4$.
- Show that the lines $2x + y - 4 = 0 = y + 2z$ and $x + 3z - 4 = 0$, $2x + 5z - 8 = 0$ are coplanar.
- Find the length and equations to the line of the shortest distance between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $x + y + 2z - 3 = 0 = 2x + 3y + 3z - 4$.

UNIT-III

- Find the equation of the sphere through (0, 0, 0), (a, 0, 0), (0, b, 0), (0, 0, c) in A, B, C. Prove that the centroid of the ΔABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$
- Find the equation of the sphere circumscribing the tetrahedron whose faces are $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
- Find the equation of the sphere passing through the circle $x^2 + y^2 = 4, z = 0$ and is intersected by the plane $x + 2y + 2z = 0$ in a circle of radius 3.
- Show that the two circles $x^2 + y^2 + z^2 - y + 2z = 0, x - y + z = 2$; $x^2 + y^2 + z^2 + x - 3y + z - 5 = 0$, $2x - y + 4z - 1 = 0$ lie on the same sphere.

UNIT-IV

- Find the power of the point (1, -1, 2) with respect to the sphere

$$x^2 + y^2 + z^2 + 2x - 4y + 6z - 5 = 0$$

15. Find the polar plane of the point (0, -1, 1) with respect to the sphere

$$x^2 + y^2 + z^2 - 2x + 4y + 6z - 11 = 0.$$

16. Find the limiting points of the coaxial system of spheres determined by

$$x^2 + y^2 + z^2 + 4x - 2y + 2z + 6 = 0, \quad x^2 + y^2 + z^2 + 2x - 4y + 2z + 6 = 0$$

17. If r_1 and r_2 are the radii of the orthogonal spheres, then show that the radius of the circle of their intersection is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$.

18. Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at (1, -2, 1) and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.

UNIT-V

19. Show that the general equation of the cone of the second degree which pass through Coordinate axes is $fyz + gzx + hxy = 0$.

20. Find the equations of the cone touches the 3 co-ordinate planes and the planes

$$x + 2y + 3z = 0, \quad 2x + 3y + 4z = 0.$$

21. Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$ with its vertex at (1, 1, 1).

22. Find the equation to the right circles cone whose vertex in P(2, -3, 5) axis PQ which makes equal angles with the axes and which passes through (1, -2, 3).

23. Find the reciprocal cone of $9x^2 + 4y^2 - 7z^2 = 0$.

Eassy Quasitions

UNIT-I

1. variable plane is at a constant distance $3p$ from the origin and meets the axes in A, B, C. Show that the locus of the centroid of the ΔABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.
2. A variable plane makes intercepts on the axes, the sum of whose squares is k^2 (a constant). Show that the locus of the foot of the perpendicular from origin to the plane is $(x^{-2} + y^{-2} + z^{-2})(x^2 + y^2 + z^2)^2 = k^2$.
3. Find the planes bisecting the angles between the planes $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$. Point out which of the planes bisects the acute angle and which bisects the obtuse angle in which the origin lies.
4. Show that the plane $8x - 14y - 13 = 0$ bisects the obtuse angle between the plane $4x + 3y - 5z + 1 = 0$ and $12x + 5y - 13z = 0$.
5. Show that the equation $x^2 + 4y^2 + 9z^2 - 12yz - 6zx + 4xy + 5x + 10y - 15z + 6 = 0$ represents a pair of parallel planes and find the distance between them.

UNIT-II

6. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$; $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.
7. Prove that the points (1, 2, 3), (4, 0, 4), (-2, 4, 2), (7, -2, 5) are collinear.
8. Find the image of the point (2, -1, 3) in the plane $3x - 2y + z = 9$.
9. Find the length and equations of shortest distance between the lines $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.
10. Find the length and equation of the shortest distance between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ and $x + y + 2z - 3 = 0 = 2x + 3y + 3z - 4$.

UNIT-III

11. A sphere of constant radius r passes through the Origin O and cuts the axes in A, B, C . Prove that the foot of perpendicular from O to the plane \overline{ABC} lies on
- $$(x^2 + y^2 + z^2)^2 (x^{-2} + y^{-2} + z^{-2}) = 4r^2$$
12. Show that the four points $(-8, 5, 2), (-5, 2, 2), (-7, 6, 6), (-4, 3, 6)$ are concyclic.
13. Show that the two circles $x^2 + y^2 + z^2 - y + 2z = 0, x - y + z = 2; x^2 + y^2 + z^2 + x - 3y + z - 5 = 0, 2x - y + 4z - 1 = 0$ lie on the same sphere.

UNIT-IV

13. Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at $(1, -2, 1)$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.
14. Show that the spheres through the origin and each set of points where the planes parallel to the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ cut the axes, form a system of spheres which are cut orthogonally by the sphere $x^2 + y^2 + z^2 + 2fx + 2gy + 2hz = 0$ if $af + bg + ch = 0$.
15. Find the limiting points of the coaxial system of spheres determined by
- $$x^2 + y^2 + z^2 + 4x - 2y + 2z + 6 = 0, x^2 + y^2 + z^2 + 2x - 4y + 2z + 6 = 0$$
16. show that radical line for the spheres $x^2 + y^2 - 4x - 2y + 3 = 0, x^2 + y^2 + z^2 - 6y + 3 = 0, x^2 + y^2 + z^2 + 4x + 2y - 4z + 3 = 0, x^2 + y^2 + z^2 + 4x + 2y - 4z + 3 = 0$ is $\frac{x}{3} = \frac{y}{2} = \frac{z}{7}$.
17. Find the radical centre of the spheres $x^2 + y^2 + z^2 + 4y = 0, x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0, x^2 + y^2 + z^2 + 3x - 2y + 8z + 6 = 0, x^2 + y^2 + z^2 - x + 4y - 6z - 2$.

UNIT-V

18. Prove that the equation $2y^2 + 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone whose vertex is $(\frac{-7}{6}, \frac{1}{2}, \frac{5}{6})$.
19. Prove that if the angle between the lines of intersection of the plane $x + y + z = 0$ and the cone $ayz + bzx + cxy = 0$ is $\pi/2$, then $a + b + c = 0$ and is $\pi/3$, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.
20. Show that two lines of intersection of the plane $ax + by + cz = 0$ with the cone $yz + zx + xy = 0$ will be perpendicular if $1/a + 1/b + 1/c = 0$.
21. Find the angle between the lines of section of the plane $3x + y + 5z = 0$ and the cone $6yz - 2zx + 5xy = 0$.
22. If $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone then prove that
- $$\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$$
22. Show that the general equation to a cone which touches the three co ordinate planes is
- $$\sqrt{ax} + \sqrt{by} + \sqrt{cz} = 0$$
23. The semi-vertical angle of a right circular cone having three mutually perpendicular
- (i) generators is $\tan^{-1}(\sqrt{2})$
- (ii) tangent planes is $\tan^{-1}(\frac{1}{\sqrt{2}})$

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA

II B.Sc. – MATHEMATICS – Semester - III (w.e.f. 2017-2018)

Course: ABSTRACT ALGEBRA

Total Hrs. of Teaching-Learning: 90 @ 6 h / Week

Total Credits: 05

Objective:

- To learn about the basic structure in Algebra
- To understand the concepts and able to write the proofs to theorems
- To know about the applications of group theory in real world problems

Unit I: Groups

(20 hours)

Binary Operation – Algebraic structure – semi group – monoid – Definition and elementary properties of a Group – Finite and Infinite groups – Examples – Order of a group – Composition tables with examples.

Unit II: Subgroups, Cosets and Lagrange's Theorem

(20hours)

Definition of Complex – Multiplication of two complexes – Inverse of a complex – Subgroup definition – Examples - Criterion for a complex to be a subgroup – Criterion for the product of two subgroups to be a subgroup – Union and intersection of subgroups.

Cosets definition – Properties of cosets – Index of subgroup of a finite group – Lagrange's Theorem.

Unit III: Normal Subgroups

(17 hours)

Definition of normal subgroup – Proper and improper normal subgroup – Hamilton group – Criterion for a subgroup to be a normal subgroup – Intersection of two normal subgroups – Subgroup of index 2 is a normal subgroup – Simple group – Quotient group – Criteria for the existence of a quotient group.

Unit IV: Homomorphism

(16 hours)

Definition of homomorphism – Image of homomorphism – elementary properties of homomorphism – Definition and elementary properties of Isomorphism and automorphism – Kernel of a homomorphism – Fundamental theorem on homomorphism and applications.

Unit V: Permutations and Cyclic Groups

(17 hours)

Definition of permutation – Permutation multiplication – Inverse of a permutation – Cyclic permutations – Transposition – Even and odd permutations – Cayley's theorem.

Definition of cyclic group - Elementary properties – Classification of cyclic groups.

Additional Inputs: Applications of group theory

Text Book:

Abstract Algebra by J.B.Fraleigh

Books for reference:

1. A text book of Mathematics, S. Chand and Company, Ltd.
2. Modern Algebra by Gupta and Malik.
3. Elements of Real Analysis by Santhi Nararayana & M. D. Raisinghania.

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-III

Unit	TOPIC	V.S.A.Q	S.A.Q	E.Q	Marks allotted to the Unit
I	Groups	1	1	2	22
II	Subgroups, Cosets & Lagrange's theorem	1	1	2	22
III	Normal Subgroups	1	1	1	14
IV	Homomorphism	1	1	1	14
V	Permutations and Cyclic groups	1	1	2	22
Total		5	5	8	94

V.S.A.Q. = Very short answer questions (1 mark)

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (8 marks)

Very short answer questions : $5 \times 1 = 05$

Short answer questions : $3 \times 5 = 15$

Essay questions : $5 \times 8 = 40$

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Total Marks = 60

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P.R. Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - III Semester
Mathematics Course: Abstract Algebra
Paper III (Model Paper w.e.f. 2018 - 2019)

Time: 2Hrs 30 min

Max. Marks: 60

PART - I

Answer the following questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Write the Cauchy's composition table for $G = \{1, \omega, \omega^2\}$.
2. Write a proper subgroup of a group $G = \{1, -1, i, -i\}$ with respect to multiplication.
3. Define normal subgroup.
4. Check whether $f: (Z, +) \rightarrow (Z, +)$ defined by $f(x) = x^2$ is a homomorphism or not.
5. Write the inverse of the permutation $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 2 & 5 \end{pmatrix}$.

PART-II

Answer any THREE questions. Each question carries 5 marks.

3 x 5 = 15 M

6. Prove that the set Z of all integers form an abelian group w.r.t. the operation defined by $a * b = a + b + 2, \forall a, b \in Z$.
7. Prove that a non empty complex H of a group G is a subgroup of G if and only if $H = H^{-1}$.
8. If M, N are two normal subgroups of G such that $M \cap N = \{e\}$ then every element of M commutes with every element of N .
9. If f is a homomorphism of a group G into a group G' , then prove that the kernel of f is a normal subgroup of G .
10. Express the product $(2 \ 5 \ 4)(1 \ 4 \ 3)(2 \ 1)$ as a product of disjoint cycles and find its inverse.

PART-III

Answer any FIVE questions from the following by choosing at least TWO from each section.

Each question carries 8 marks.

5 x 8 = 40 M

SECTION-A

11. Show that the n^{th} roots of unity form an abelian group with respect to multiplication.
12. Prove that a semi group (G, \cdot) is a group if and only if the equations $ax = b, ya = b \forall a, b \in G$ have unique solutions in G .
13. State and Prove the necessary and sufficient condition for a finite complex H of a group G to be a subgroup of G .
14. Prove that the union of two subgroups of a group ' G ' is a subgroup of ' G ' if and only if one is contained in the other.

SECTION-B

15. If H is a normal subgroup of a group (G, \cdot) , then prove that the product of two right (left) cosets of H is also a right (left) coset of H .
16. Prove that every homomorphic image of a group G is isomorphic to some quotient group of G .
17. Prove that the set A_n of all even permutations form a normal subgroup of the group of permutations S_n .
18. Prove that every subgroup of a cyclic group is cyclic.

PR GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank

PAPER-III: ABSTRACT ALGEBRA

Very Short Answer Questions

UNIT-1

1. Define algebraic structure and give one example.
2. Define semi group and give one example.
3. Define monoid and give one example.
4. Define group and give one example.
5. Define order of a group and give one example.
6. Define cancellation laws in a group.
7. Is $(\mathbb{Z}, +)$ a group and justify your answer.
8. Find the identity element in the set Q^+ of all positive rational numbers with respect to the operation \circ defined by $a \circ b = \frac{ab}{3}, \forall a, b \in Q^+$.
9. Find the order of each element of the multiplicative group $G = \{1, -1, i, -i\}$.
10. In a group $G, a^2 = e, \forall a \in G$, then prove that G is abelian.
11. Define complex of a group and give example

UNIT-II

12. Define subgroup of a group.
13. Is the complex $(\mathbb{Z} - \{0\}, \cdot)$ of the group $(\mathbb{R} - \{0\}, \cdot)$ is a subgroup of $(\mathbb{R} - \{0\}, \cdot)$?
14. If $G = \{1, -1, i, -i\}$ is a group under multiplication, then write all the subgroups of G .
15. Define left and right cosets.
16. If (G, \cdot) is an abelian group then what is the Normalizer of an element of $a \in G$.
17. State Lagrange's theorem.

UNIT-III

18. Define simple group
19. Define normal subgroup
20. Define maximal normal subgroup of a group
21. Define index of a subgroup of a group
22. Write the quotient group of $H = \{0, 3, 6, 9, 12\}$ in $(\mathbb{Z}_{15}, +_{15})$.

UNIT-IV

23. Define
 - a. homomorphism,
 - b. homomorphic image,
 - c. one-one (injective) mapping,
 - d. onto (surjective) mapping,
 - e. mono morphism,
 - f. epi morphism,
 - g. isomorphism,
 - h. automorphism.
24. If $f: \mathbb{Z} \rightarrow \mathbb{Z}$ is defined as $f(n) = 2n$ then find $\ker f$.
25. State fundamental theorem of Homomorphism of groups.

UNIT-V

26. If $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ & $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ then find fg .
27. Define Transposition
28. Define even and odd permutations.
29. Express $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 4 & 6 & 5 \end{pmatrix}$ as a product of transpositions.
30. Define Cyclic group.
31. Find the generators of $(\{1, \omega, \omega^2\}, X)$
32. If (G, \cdot) is a group of order 48 then how many generators G has?

Short Answer Questions

UNIT-1

1. Prove that in a group the identity element is unique and the inverse of every element is unique.
2. Prove that the cancellation laws hold in a group.
3. Prove that a semi group satisfying cancellation laws is a group.
4. Show that a group (G, \cdot) is abelian if and only if $(ab)^2 = a^2b^2 \forall a, b \in G$.
5. If (G, \cdot) is a group such that $(ab)^n = a^n b^n, \forall a, b \in G$ for three consecutive positive integers n , then show that (G, \cdot) is an abelian group.
6. Show that the fourth roots of unity form an abelian group under multiplication.
7. In a group G , if $a \in G$ then prove that $O(a) = O(a^{-1})$.

UNIT-II

8. If H and K are two subgroups of a group G then show that $H \cap K$ is also a subgroup of G .
9. Prove that a non empty complex H of a group G is a subgroup of G if and only if $H = H^{-1}$.
10. Prove that the union of two subgroups of a group is a subgroup if and only if one is contained in the other.
11. Prove that a non empty finite complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab \in H$.
12. $(Z_6 = \{0,1,2,3,4,5\}, +_6)$ is a group. Prove that $S = \{0,2,4\}, T = \{0,3\}$ are subgroups of Z_6 and $S \cup T$ is not a subgroup of Z_6 .

UNIT-III

13. If G is a group and $a \in G$, then prove that the normalizer $N(a) = \{x \in G / ax = xa\}$ is a subgroup of G .
14. Prove that any two left cosets of a subgroup H in a group (G, \cdot) are either disjoint or identical.
15. State and Prove Lagrange's theorem.
16. If M, N are two normal subgroups of G such that $M \cap N = \{e\}$ then every element of M commutes with every element of N .
17. Define Normal subgroup. Prove that a subgroup H of a Group (G, \cdot) is a normal subgroup of G if and only if $xHx^{-1} = H \forall x \in G$.
18. Prove that a subgroup H of a Group (G, \cdot) is a normal subgroup of G if and only if every left coset of H in G is a right coset of H in G .
19. Prove that a subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G .

20. Prove that every subgroup of an abelian group is normal.
21. If G is a group and H is a subgroup of index 2 in G , then prove that H is a normal subgroup of G .
22. If G is a group then prove that the centre Z of G is a normal subgroup of G .

UNIT-IV

23. Let a be a fixed element of a group G then prove that the mapping $f_a: G \rightarrow G$ defined by $f_a(x) = a^{-1}xa$ for every $x \in G$ is an automorphism.
24. If f is a homomorphism from a group G to a group G' then prove that $f(a^{-1}) = [f(a)]^{-1}$.
25. Let G be a group. If $f: G \rightarrow G$ defined by $f(x) = x^2 \forall x \in G$ is a homomorphism then show that G is abelian.
26. Show that the group $(G = \{0,1,2,3\}, +_4)$ and the group $(G' = \{1, -1, i, -i\}, \cdot)$ are isomorphic

UNIT-V

27. $f = (1\ 2\ 3\ 4\ 5\ 6\ 7\ 8)$, $g = (4\ 1\ 5\ 6\ 7\ 3\ 2\ 8)$ are cyclic permutations. Show that $(fg)^{-1} = g^{-1}f^{-1}$.
28. Write down the following permutation as product of disjoint cycles
 $f = (1\ 3\ 2\ 5)(1\ 4\ 3)(2\ 5\ 1)$.
29. Find the regular permutation group isomorphic to the multiplicative group $G = \{1, -1, i, -i\}$.
30. Show that every cyclic group is an abelian group.
31. Show that the group $(G = \{1,2,3,4,5,6\}, X_7)$ is a cyclic group and write down all of its generators.
32. Find the number of generators of cyclic groups of orders 5, 6, 8, 12.

Essay Questions

UNIT-1

1. Prove that a semi group (G, \cdot) is a group if and only if the equations $ax = b, ya = b \forall a, b \in G$ have unique solutions in G .
2. Show that the set of integers is an abelian group with respect to 'o' where 'o' is defined as $aob = a + b + 1 \forall a, b \in \mathbb{Z}$.
3. Prove that the set Z of all integers form an abelian group w.r.t. the operation is defined by $a * b = a + b + 2 \forall a, b \in \mathbb{Z}$
4. Prove that the set $G = \mathbb{Q} - \{1\}$ is an abelian group with respect to 'o' defined by $aob = a + b - ab \forall a, b \in G$.
5. Prove that the set of n^{th} roots of unity forms an abelian group w.r.t. ' \cdot '
6. Prove that the set $G = \{A_\alpha / \alpha \in \mathbb{R}\}$ where $A_\alpha = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$ forms a group with respect to ' \cdot '.

UNIT-II

7. Prove that a non empty complex H of a group G is a subgroup of G if and only if (i) $a, b \in H \Rightarrow ab \in H$ (ii) $a \in H \Rightarrow a^{-1} \in H$.
8. Prove that a non empty complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$.
9. Prove that a non empty finite complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab \in H$.
10. State and prove the Necessary and sufficient condition for a finite complex H of a group G .
11. Prove that the union of two subgroups of a group is a subgroup if and only if one is contained in the other.
12. If N is a normal subgroup of a group G . Show that G/N is abelian if and only if $x, y \in G, xyx^{-1}y^{-1} \in N$.
13. State and prove Lagrange's Theorem. Prove that the converse of Lagrange's theorem is not true.
14. Prove that $(\{0,3,6,9,12\}, +_{15})$ is a subgroup of $(Z_{15}, +_{15})$. Find the left cosets of the above subgroup in Z_{15} . Find the index of subgroup in Z_{15} .

UNIT-III

15. If H is a subgroup of G and N is a normal subgroup of G , then prove that
(i) $H \cap N$ is a normal subgroup of H (ii) N is a normal subgroup of HN .
16. If H is a normal subgroup of a group (G, \cdot) then prove that the product of two right (or) left cosets of H is also a right (or) left coset of H in G .
17. If H is a normal subgroup of G . The set $\frac{G}{H}$ of all cosets of H in G with respect to coset multiplication is a group.
18. If N, M are normal subgroups of G , then prove that NM is also a normal subgroup of G .

UNIT-IV

19. Prove that the homomorphic image of a group G is isomorphic to some quotient group of G .
20. $(Z, +)$ is a group of integers. Show that $f: Z \rightarrow Z$ defined by $f(x) = 2x \forall x \in Z$ is a homomorphism. Find $\text{Ker } f$ and show that it is not an onto homomorphism.
21. Let (R^+, \cdot) be a group and $f: R^+ \rightarrow R^+$ defined by $f(x) = x^2, \forall x \in R^+$. Prove that f is an automorphism.
22. Show that if $f: G \rightarrow G'$ defined by $f(a) = a^{-1} \forall a \in G$ is an automorphism if and only if G is abelian.

UNIT-V

23. $f = (1\ 2\ 3\ 4\ 5\ 6\ 7\ 8), g = (4\ 1\ 5\ 6\ 7\ 3\ 2\ 8)$ are cyclic permutations. Show that $(fg)^{-1} = g^{-1}f^{-1}$.
24. Prove that the set A_n of all even permutations form a normal subgroup of the group of permutations S_n .
25. State and prove Cayley's theorem.
26. Prove that every subgroup of a cyclic group is cyclic.
27. Prove that a cyclic group of order 'n' has $\phi(n)$ generators.
28. Prove that the set of n^{th} roots of a unity form a cyclic group.

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
II B.Sc. MATHEMATICS-Semester IV (w.e.f. 2017-2018)

Course: Real Analysis

Total Hrs. of Teaching-Learning: 90 @ 6 hr/Week

Total credits: 5

OBJECTIVES:

- Be able to understand and write clear mathematical statements and proofs.
- Be able to apply appropriate method for checking whether the given sequence or series is convergent.
- Be able to develop students ability to think and express themselves in a clear logical way.
- This curriculum gives an opportunity to learn about the derivatives of functions and its applications.

Unit I: Real Number System and Real Sequence (18 hours)

The algebraic and order properties of \mathbb{R} – Absolute value and Real line – Completeness property of \mathbb{R} – Applications of supreme property – Intervals - Limit point of a set - Existence of limit points. (No questions to be set from this portion)

Sequences and their limits – Range and Boundedness of sequences - Necessary and sufficient condition for convergence of Monotone sequence, limit point of a sequence, Subsequences and the Bolzano Weierstrass theorem - Cauchy sequences – Cauchy's general principle of convergence theorems.

Unit II: Infinite Series (18 hours)

Introduction to Infinite Series – Convergence of series – Cauchy's general principle of convergence for series – Tests for convergence of non negative terms – P - test – Limit comparison test – Cauchy's nth root test – De-Alambert's ratio test - Alternating series – Liebnitz's test - Absolute and conditional convergence.

Unit III: Limits and Continuity (18 hours)

Real valued functions – Boundedness of a function - Limit of a Function, One-sided Limits- Right hand and Left Hand Limits - Limits at Infinity - Infinite Limits. (no question to be set)

Continuous Functions - Discontinuity of a Function - Algebra of Continuous Functions – Continuous functions on intervals - Some Properties of Continuity of a function at a point - Uniform Continuity.

Unit IV: Differentiation and Mean Value Theorem (18 hours)

The Derivability of a function, on an interval, at a point, Derivability and Continuity of a function - Geometrical meaning of the Derivative - Mean Value Theorems - Rolle's Theorem, Lagrange's Mean Value theorem, Cauchy's Mean Value theorem.

Unit V: Riemann Integration (18 hours)

Riemann sums, Upper and Lower Riemann integrals, Riemann integral, Riemann Integrable function – Darboux's Theorem - Necessary and sufficient conditions for Riemann integrability – properties of integrable functions – Fundamental Theorem of Integral Calculus – Integral as the limit of a sum – Mean Value Theorems.

Additional Inputs :

1. Problems using Cauchy's first theorem on limits and Cauchy's second theorem on limits.
2. Statement of Maclaurin's theorem and expansions of e^x , $\sin x$, $\cos x$, $\log(1+x)$.

Prescribed book:

- Real Analysis by Rabert & Bartely and D. R. Sherbart, published by John Wiley.

Reference books:

- Elements of Real Analysis by Santhi Narayanan & M. D. Raisinghania, published by S.Chand & Company Pvt. Ltd., New Delhi.
- Course on Real analysis by N. P. Bali - Golden series publications.
- A Text Book of Mathematics Semester IV by V. Venkateswarrao & others, published by S.Chand & Company Pvt. Ltd., New Delhi

**BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-IV**

Unit	TOPIC	V.S.A.Q	S.A.Q	E.Q	Marks allotted
I	Real Number System and Real Sequence	1	1	1	14
II	Infinite Series	1	1	2	22
III	Limits and Continuity	1	1	1	14
IV	Differentiation and Mean Value Theorem	1	1	2	22
V	Riemann Integration	1	1	2	22
	TOTAL	5	5	8	94

V.S.A.Q. = Very short answer questions (1 mark)

S.A.Q. = Short answer questions (5 marks)

E.Q. = Essay questions (8 marks)

Very short answer questions : $5 \times 1 = 05$

Short answer questions : $3 \times 5 = 15$

Essay questions : $5 \times 8 = 40$

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Total Marks = 60
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P.R. Government College (Autonomous), Kakinada
II Year, B.Sc., Degree Examinations - IV Semester
Mathematics Course: Real Analysis
Paper-IV (Model Paper w. e. f. 2018-2019)

Time: 2Hrs 30 min

Max. Marks: 60

PART-I

Answer ALL the questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Define convergence of a sequence.
2. Test the convergence of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$.
3. Define continuity of a function at a point.
4. Give an example of a function which is continuous but not derivable.
5. Evaluate $\lim_{n \rightarrow \infty} \frac{1}{n} [e^{\frac{3}{n}} + e^{\frac{6}{n}} + e^{\frac{9}{n}} + \dots + e^{\frac{3n}{n}}]$.

PART-II

Answer any THREE questions. Each question carries 5 marks.

3 x 5 = 15 M

6. Show that $\lim_{n \rightarrow \infty} \left[\sqrt{\frac{1}{n^2+1}} + \sqrt{\frac{1}{n^2+2}} + \dots + \sqrt{\frac{1}{n^2+n}} \right] = 1$.
7. State and Prove Leibnitz's test.
8. Examine for continuity the function f defined by $f(x) = |x| + |x - 1|$ at 0 and 1.
9. Show that every derivable function on a closed interval is continuous.
10. State and prove fundamental theorem of Integral Calculus.

PART-III

Answer any FIVE questions from the following by choosing at least TWO from each section.

Each question carries 8 marks.

5 x 8 = 40 M

SECTION - A

11. Prove that every monotonically increasing sequence which is bounded above converges to its least upper bound.
12. State and Prove Cauchy's n^{th} root test for the convergence of series.
13. Examine the convergence of $\sum_{n=1}^{\infty} (\sqrt{n^3+1} - \sqrt{n^3})$.
14. Prove that every continuous function is bounded and attains its bounds.

SECTION - B

15. State and prove Rolle's theorem.
16. Using Lgrange's mean value theorem, prove that $1+x < e^x < 1+xe^x$, $\forall x > 0$.
17. Prove that $f(x) = \sin x$ is integrable on $[0, \frac{\pi}{2}]$ and $\int_0^{\frac{\pi}{2}} \sin x \, dx = 1$.
18. State the first mean value theorem of integral calculus and by using it prove that

$$\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3 \cos x} \, dx \leq \frac{\pi^3}{6}.$$

PR GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank
PAPER-IV: REAL ANALYSIS

Very Short Answer Questions

UNIT-I

1. Define convergence of a sequence.
2. Define limit of a sequence.
3. Give an example of a sequence which is bounded but not convergent.
4. Show that $\left\{\frac{1}{2^n}\right\}$ is a subsequence of $\left\{\frac{1}{n}\right\}$.
5. Find $\lim_{n \rightarrow \infty} \frac{1^3+2^3+\dots+n^3}{n^4}$.
6. Show that the sequence $\left\{\frac{1}{n}\right\}$ converges to 0
7. State the Monotone Convergence theorem.
8. Define Cauchy sequence.
9. Find the range set and limit points of the sequence $\{S_n\}$ defined by $S_n = 1 + (-1)^n$
 $\forall n \in N$.

UNIT-II

10. State Geometric series test.
11. Test the convergence of $\sum \frac{1}{3^{n-1}}$
12. Test the convergence of $\sum_{n=1}^{\infty} \log\left(\frac{1}{n}\right)$.
13. Test the convergence of $\sum \frac{2^n}{n^3}$.
14. Test for convergence $\sum \left(\frac{n+1}{n+2}\right)^n \cdot x^n \forall x > 0$.
15. Test the convergence of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$

UNIT -III

16. Define limit of a function at a point.
17. Prove that $\lim_{x \rightarrow \infty} \frac{3x+|x|}{7x-5|x|}$ does not exist
18. Define continuity of a function at a point.
19. Define continuity of a function on an interval
20. Define uniform continuity of a function

UNIT-IV

21. Define derivability of a function on an Interval
22. Give an example of a function which is continuous but not derivable.
23. Show that $f(x) = \sin x$ is derivable at every $a \in R$
24. Find the interval in which the function $f(x) = x^2 - 4x + 1$ is increasing.
25. State Rolle's theorem
26. State Lagrange's mean value theorem
27. State Cauchy's mean value theorem

UNIT-V

28. Define upper and lower Riemann sums
29. Write the Necessary and sufficient condition for integrability.
30. State fundamental theorem of integral calculus

31. Show that $\int_0^1 x^4 dx = \frac{1}{5}$
19. Evaluate $\lim_{n \rightarrow \infty} \frac{1}{n} [e^{\frac{3}{n}} + e^{\frac{6}{n}} + e^{\frac{9}{n}} + \dots + e^{\frac{3n}{n}}]$.

Short Answer Questions

UNIT-I

1. By the definition of limit of a sequence show that $\lim \sqrt[n]{n} = 1$.
2. Show that $\lim_{n \rightarrow \infty} \left[\sqrt{\frac{1}{n^2+1}} + \sqrt{\frac{1}{n^2+2}} + \dots + \sqrt{\frac{1}{n^2+n}} \right] = 1$.
3. State and prove sandwich theorem
4. Prove that every convergent sequence is a Cauchy sequence
5. Prove that if $\{S_n\}$ is a Cauchy sequence, then $\{S_n\}$ is bounded.
6. Show that the sequence $\{a_n\}$ defined by $a_n = \left(1 + \frac{1}{n}\right)^n$ is convergent.

UNIT-II

7. State and prove cauchy's general principle of convergence for series.
8. Test for convergence of $\sum \frac{1}{2^n+3^n}$
9. Examine the convergence of $\sum_{n=1}^{\infty} (\sqrt{n^3+1} - \sqrt{n^3})$.
10. Examine the convergence of $\sum_{n=1}^{\infty} (\sqrt[3]{n^3+1} - n)$.
11. Test for the convergence of $\sum_{n=1}^{\infty} \frac{1.3.5 \dots (2n-1)}{2.4.6 \dots 2n} x^{n-1} (x > 0)$

UNIT-III

12. Examine the continuity of the function defined by $f(x) = |x| + |x - 1|$ at $x = 0, 1$.
13. Find the points of the discontinuity of $f(x) = \frac{1}{2^n}$ for $\frac{1}{2^{n+1}} < x \leq \frac{1}{2^n}$, where $n = 0, 1, 2, \dots$ and $f(0) = 0$
14. If $f: [a, b] \rightarrow R$ is continuous on $[a, b]$, then prove that f is bounded on $[a, b]$. Is the converse true and justify your answer
15. Define uniform continuity and prove that every uniform continuous function is continuous.
16. Prove that $f: R \rightarrow R$ given by $f(x) = x^2$ is a continuous function on R , but not uniformly continuous on R .
17. Prove that a function defined on $[a, b]$ is uniformly continuous on $[a, b]$ if it is continuous on $[a, b]$

UNIT-IV

18. If $f: [a, b] \rightarrow R$ is derivable at $c \in [a, b]$, then prove that f is continuous on $[a, b]$. Is its converse true? Justify your answer.
19. Test the continuity and differentiability of $f(x) = x \left(\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} \right)$ if $x \neq 0$ and $f(0) = 0$ at $x = 0$.
20. Show that $f(x) = x \sin(1/x), x \neq 0; f(x) = 0, x = 0$ is continuous but not derivable at $x = 0$.
21. State and Prove Rolle's theorem.
22. Find c of Cauchy's mean value theorem for $f(x) = \sqrt{x}, g(x) = \frac{1}{\sqrt{x}}$ in $[a, b]$ where $0 < a < b$.
23. Examine the applicability of Rolles theorem for $f(x) = 1 - (x - 1)^{2/3}$ on $[0, 2]$.
24. Verify Lagranges Mean Value theorem for $f(x) = (x-1)(x-2)(x-3)$ on $[0, 4]$.
25. Using LMVT prove that $1 + x < e^x < 1 + xe^x, \forall x > 0$

UNIT-V

26. If $f(x) = x^2 \forall x \in [0,1]$ and $P = \{0, 1/4, 2/4, 3/4, 1\}$, then find $U(P, f)$ and $L(P, f)$.
27. If f is continuous on the $[a, b]$ then prove that f is Riemann Integrable on the $[a, b]$.
28. If f is monotonic on the $[a, b]$ then prove that f is Riemann Integrable on the $[a, b]$.
29. Show that the function f defined by $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$ for all x in $[0,1]$ is not R-integrable on $[0,1]$.
30. Using 1st mean value theorem of integral calculus show that $x > \log(1+x) > \frac{x}{1+x}$, $x \geq 0$

Essay Questions

UNIT-I

1. Prove that the sequence $\{s_n\}$ defined by $s_1 = \sqrt{c} > 0, s_{n+1} = \sqrt{c + s_n}$ for all $n \in \mathbb{Z}^+$ converges to the positive root of $x^2 - x - c = 0$.
2. State and Prove monotone convergence theorem.
3. Prove that the sequence $\{s_n\}$ defined by $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!}$ is convergent.
4. Show that the sequence $\{a_n\}$ defined by $a_n = \left(1 + \frac{1}{n}\right)^n$ is convergent.
5. Prove that every bounded sequence has at least one limit point
6. State and prove cauchy's convergence theorem on sequences

UNIT -II

7. State and prove Limit comparison test.
8. State and prove cauchy's n^{th} root test .
9. Test for convergence of $\sum_{n=1}^{\infty} \sqrt[3]{n^3} + 1 - n$.
10. State and prove D'Alemberts ratio test.
11. State and prove Leibnitz test
12. Prove that $\sum_{n=1}^{\infty} (-1)^n \sin \frac{1}{n}$ is not absolutely convergent.
13. Examine the convergence of $\sum_{n=1}^{\infty} (\sqrt{n^4 + 1} - \sqrt{n^4 - 1})$.

UNIT-III

14. If $f: [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$, then f is bounded on $[a, b]$ and attains its bounds or infimum and supremum
15. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be such that $f(x) = \frac{\sin(a+1)x + \sin x}{x}$ for $x < 0, f(x) = c$ for $x = 0$ and

$f(x) = \frac{(x+bx^2)^{-\frac{1}{3}} - x^{\frac{1}{2}}}{bx^{\frac{3}{2}}}$ for $x > 0$. Determine the values of a, b, c which the function is continuous at $x = 0$

16. Test the continuity of $f(x) = \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$ if $x \neq 0$ and $f(0) = 0$ at $x = 0$.
16. If f is a continuous function on $[a, b]$, then prove that it is uniformly continuous on $[a, b]$.
17. State and Prove Intermediate value theorem.

UNIT-IV

18. State and Prove Lagrange's theorem and write the geometrical interpretation of Lagrange's theorem.
19. Discuss the applicability of Lagrange's mean value theorem for $f(x) = x(x-1)(x-2)$ on $[0, 1/2]$.
20. Using Lagrange's mean value theorem, show that $x > \log(1+x) > \frac{x}{1+x}$ if $f(x) = \log(1+x) \forall x > 0$
21. Show that $x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$, $\forall x > 0$
22. Show that $\frac{v-u}{1+v^2} < \tan^{-1}v - \tan^{-1}u < \frac{v-u}{1+u^2}$ for $0 < u < v$.

Hence deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

23. Prove that $\frac{\pi}{6} + \frac{\sqrt{3}}{15} < \sin^{-1}0.6 < \frac{\pi}{6} + \frac{1}{8}$
24. State and Prove Cauchy's mean value theorem.
25. Find c of Cauchy's mean value theorem for $f(x) = \sqrt{x}, g(x) = \frac{1}{\sqrt{x}}$ in $[a, b]$ where $0 < a < b$.

UNIT-V

26. State and prove Necessary and Sufficient condition for integrability.
27. Show that (i) $\int_{-2}^2 |x| dx = 4$, (ii) $\int_{-1}^4 x|x| dx = \frac{2}{3}$
28. Prove that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3 \cos x} dx \leq \frac{\pi^3}{6}$.
29. Show that $\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2} dx \leq \frac{2}{\pi}$
30. State and prove fundamental theorem of Integral Calculus.
31. State and Prove first mean value theorem of integral calculus.
32. Using 1st mean value theorem of integral calculus show that $x > \log(1+x) > \frac{x}{1+x}$, $x \geq 0$

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
II B.Sc. /B.A. /B.Com. - Semester IV (w.e.f 2017-2018)
Course: ANALYTICAL SKILLS

Total Hrs. of Teaching-Learning: 30@ 2 hr/Week

Total credits: 02

Objectives:

- To impart the knowledge of arithmetic and reasoning.
- To built up confidence for writing competitive examinations.

UNIT – 1: Data Analysis

(6 hrs)

The data given in a Table – Graph - Bar Diagram - Pie Chart - Venn diagram or a Passage is to be analyzed and the questions pertaining to the data are to be answered.

UNIT – 2: Sequence and Series

(6 hrs)

Analogies of numbers and Alphabets - Completion of blank spaces following the pattern \bar{m} A:b::C: d relationship - Odd thing out - Missing number in a sequence or a series.

UNIT – 3: Arithmetic Ability

(6 hrs)

Algebraic Operations - BODMAS – Fractions - Divisibility Rules – LCM & GCD (HCF) - Date, Time and Arrangement Problems, Calendar problems, Clock problems, Blood Relationship.

UNIT – 4: Quantitative Aptitude

(6 hrs)

Averages - Ratio and Proportion - Problems on Ages – Time - Distance – Speed.

UNIT – 5: Business Computations

(6 hrs)

Percentages - Profit and Loss - Partnership - Simple and Compound Interest.

Reference Books:

1. Quantitative Aptitude for Competitive Examination by R S Agrawal, S.Chand publication s

**BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-IV
ANALYTICAL SKILLS**

UNIT	TOPIC	V.S.A.Q Multiple choice (1 Mark)	S.A.Q (3 Marks)	E.Q (5 Marks)	
1	Data Analysis	-	-	2	10
2	Sequence and Series	10	-	-	10
3	Arithmetic ability	-	3	2	19
4	Quantitative aptitude	-	2	2	16
5	Business computations	-	3	2	19
TOTAL MARKS					74

V.S.A.Q = Very short answer questions (1 mark)

S.A.Q = Short answer questions (3 marks)

E.Q = Essay questions (5 marks)

Very short answer questions : 10 X 1 = 10

Short answer questions : 05 X 3 = 15

Essay questions : 05 X 5 = 25

.....
Total Marks = 50
.....

P.R. Government College (Autonomous), Kakinada
II B.Sc./BA/B.Com. Degree Examination – Semester : IV
ANALYTICAL SKILLS
Model Paper (w.e.f. 2017-2018)

Time: 2 Hrs

Total Marks: 50M

SECTION- A

Answer all questions.

10 x 1 = 10M

అన్నీ ప్రశ్నలకు సమాధానములు వ్రాయండి.

1. Missing number in the series is _____. ()

ఈ క్రింది శ్రేణిలో ? వున్న చోట వుండే సంఖ్య _____.

1, 9, 25, 49, ?, 121

a) 64 b) 81 c) 91 d) 100

2. Missing number in the series is _____. ()

ఈ క్రింది శ్రేణిలో ? వున్న చోట సంఖ్య _____.

4, 10, 18, 28, 40, 54, 70, ?

a) 85 b) 86 c) 87 d) 88

3. The wrong number in the following series is _____. ()

ఈ క్రింది శ్రేణిలో తప్పుగా ఇవ్వబడిన సంఖ్య _____.

8, 13, 21, 32, 47, 63, 83

a) 13 b) 21 c) 32 d) 47

4. Missing alphabet in the following series is _____. ()

ఈ క్రింది శ్రేణిలో ?, ? వున్న చోట వుండే అక్షరాలు _____.

T, R, P, N, L, ?, ?

a) F b) J, H c) K, H d) K, L

5. Missing Alphabet in the following series is _____. ()

ఈ క్రింది శ్రేణిలో ? వున్న చోట వుండే అక్షరాలు _____.

AB, DEF, HIJK, ?, STUVWX

a) LMNO b) LMNOP c) MNOPQ d) QRSTU

6. Missing term in the following series is _____. ()

ఈ క్రింది శ్రేణిలో తదుపరి చివరి పదాలు _____.

D - 4, F - 6, H - 8, J - 10, ?, ?

a) K - 12, M - 13 b) L - 12, M - 14 c) L - 12, N - 14 d) K - 12, M - 14

7. Fill the missing letters in the following series. ()

ఈ క్రింది అక్షర శ్రేణిలో ఖాళీలను పూరింపుము.

__ aba __ ba _ ab

- a) abba b) abba c) baab d) bbaba

8. Find relationship between the words ()

పద సారూప్యతను అనుసరించి ఈ క్రింది ప్రశ్నలో ? ఉన్న చోట ఉండే పదం

Botany : Plants : : Entomology : ?

- a) Snakes b) insects c) Birds d) Germs

9. Find relationship between the words ()

పద సారూప్యతను అనుసరించి ఈ క్రింది ప్రశ్నలో ? ఉన్న చోట ఉండే పదం

Needle : Thread :: Pen : ?

- a) Ink b) Cap c) Paper d) Word

10. Find relation ship between the numbers is ()

సంఖ్య సారూప్యతను అనుసరించి ఈ క్రింది ప్రశ్నలో ? ఉన్న చోట ఉండే పదం

18 : 30 :: 36 : ?

- a) 54 b) 62 c) 64 d) 66

SECTION - B

Answer any FIVE of the following questions.

5 x 3 = 15 M

ఏవేని క్రింది ఐదు ప్రశ్నలకు సమాధానములు వ్రాయండి.

11. Find the value of $\frac{(6+6+6+6)+6}{4+4+4+4+4}$.
 $\frac{(6+6+6+6)+6}{4+4+4+4+4}$ విలువను కనుగొనుము.

12. If the number 517 * 324 is completely divisible by 3, then the smallest whole number in Place of * will be.

517 * 324 అనే సంఖ్య 3 చేత నిశ్శేషంగా భాగింపబడితే, * స్థానంలో ఇమిడే కనిష్ట పూర్ణాంక సంఖ్యను కనుగొనుము.

13. A is B's sister. C is B's mother. D is C's father. E is D's mother. Then, how is A related to D ?

B సోదరి A. B తల్లి C. C తండ్రి D. D తల్లి E అయితే, D తో A కు గల సంబంధం ?

13. If A : B = 2 : 3 and B : C = 4 : 7, then find A : B : C = ?

A : B = 2 : 3 మరియు B : C = 4 : 7, అయితే A : B : C ను కనుగొనుము.

15. The average of four consecutive even numbers is 27. Then, find the largest of these Numbers.

నాలుగు వరుస సరి సంఖ్యల సగటు 27. అయితే, వాటిలో గరిష్ట సంఖ్యను కనుగొనుము.

16. What is 25% of 25% equal to ?

25% యొక్క 25% దేనికి సమానం.

17. A man buys a cycle for Rs.1400 and sells it at a loss of 15%. What is the selling price of the cycle?

ఒక వ్యక్తి ఒక సైకిల్ ను రూ. 1400 కొని, 15% నష్టంతో అమ్మెను. అమ్మిన ధర ఎంత?

18. Find the simple interest on Rs 7500 in 4 years at 15%.

4 సంవత్సరాలలో 15% వడ్డీ రేటుతో 7500/- పై వచ్చే సాధారణ వడ్డీని కనుగొనుము.

SECTION - C

Answer any FIVE of the following questions.

5 x 5 = 25 M

ఏవేని క్రింది ఐదు ప్రశ్నలకు సమాధానములు వ్రాయండి.

19. DIRECTIONS: Study the table carefully to answer the questions that follow.

సూచనలు: క్రింది పట్టికను చదివి, ఆ దిగువన ఇవ్వబడిన ప్రశ్నలకు సమాధానములు వ్రాయండి.

Maximum and minimum Temperature (in degree Celsius) recorded on first day of each month for five different cities.

5 బిన్న పట్టణాలలో ప్రతి నెల 1 వ తారీఖున నమోదైన గరిష్ట మరియు కనిష్ట ఉష్ణోగ్రతలు (సెల్సియస్ లలో)

Month / నెల	Temperature / ఉష్ణోగ్రత									
	Bhuj		Sydney		Ontario		Kabul		Beijing	
	Max / గరిష్ట	Min / కనిష్ట	Max / గరిష్ట	Min / కనిష్ట	Max / గరిష్ట	Min / కనిష్ట	Max / గరిష్ట	Min / కనిష్ట	Max / గరిష్ట	Min / కనిష్ట
1 st Sep	24	14	12	2	5	1	34	23	12	9
1 st Oct	35	21	5	-1	15	6	37	30	9	3
1 st Nov	19	8	11	3	4	0	45	36	15	1
1 st Dec	9	2	-5	-9	-11	-7	31	23	2	-3
1 st Jan	-4	-7	-11	-13	-14	-19	20	11	5	-13

1. What is the difference between the max temperature of Ontario on 1st Nov and the min temperature of Bhuj on 1st Jan?

Ontario లో 1st Nov లో నమోదైన గరిష్ట ఉష్ణోగ్రతకు మరియు Bhuj లో 1st Jan లో కనిష్ట ఉష్ణోగ్రతకు గల వ్యత్యాసం

(1) 3 °C (2) 18 °C (3) 15 °C (4) 9 °C (5) 11 °C

2. In which month respectively the max temperature of Kabul is 2nd highest and min temperature of Sydney is highest ?

Kabul రెండవ గరిష్ట ఉష్ణోగ్రత మరియు Sydney మొదటి కనిష్ట ఉష్ణోగ్రత వరుసగా ఏ ఏ నెలలలో నమోదైనాయి.

- (1) 1st oct & 1st jan (2) 1st oct & 1st nov (3) 1st dec & 1st jan (4) 1st sept & 1st jan (5) 1st dec & 1st Sept

3. In which month on 1st day is the difference between the max temperature & min temperature of Bhuj second highest

Bhuj గరిష్ట మరియు కనిష్ట ఉష్ణోగ్రతలో భేదం ఏ నెల మొదటి భేదం తారీఖున రెండవ అత్యుత్తముగా ఉన్నాయి.

- (1) 1st sept (2) 1st oct (3) 1st no (4) 1st dec (5) 1st jan

4. What is the average maximum temperature of Beijing over all the months together

అన్నీ నెలల మీద Beijing గరిష్ట ఉష్ణోగ్రత సగటు ఎంత

- (1) 8.4°C (2) 9.6°C (3) 7.6°C (4) 9.2°C (5) 8.6°C

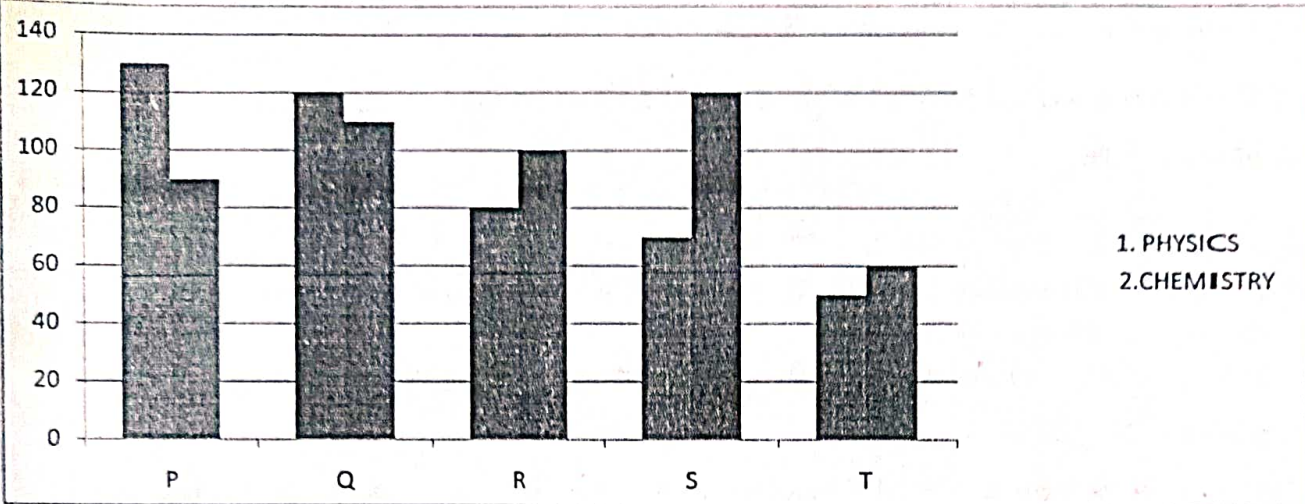
5. What is the respective ratio between the min temperature of Beijing on 1st sep & the max temperature of Ontario on 1st oct ?

1st sep లో Beijing కనిష్ట ఉష్ణోగ్రత కు, 1st oct లో Ontario గరిష్ట ఉష్ణోగ్రత కు మధ్య నిష్పత్తి ఎంత ?

- (1) 3:4 (2) 3:5 (3) 4:5 (4) 1:5 (5) 1:4

20. Study the following bar graphs carefully to answer these questions. Marks obtained by 5 students in physics & chemistry.

ఈ క్రింది బార్ చిత్రల్ని జాగ్రత్తగా చదివి, ఆ దిగువన ఉన్న ప్రశ్నలకు సమాధానములిమ్ము. Physics & Chemistry లో 5 గురు విద్యార్థులకు వచ్చిన మార్కులు.



1. Marks obtained by S in chemistry is what percent of the total marks obtained by all the students in chemistry ?

Chemistry లో అందరి విద్యార్థులకు వచ్చిన మొత్తం మార్కులలో, Chemistry లో S కు వచ్చిన మార్కుల శాతం ఎంత ?

- (1)25 (2)28.5 (3)35 (4)31.5 (5)22

2. If the marks obtained by T in physics were increased by 14% of the original marks, what would be his new approximate % in physics if the max marks in physics were 140?

Physics లో T కు వచ్చిన అసలు మార్కులకు 14% కు పెంచిన మరియు Physics లో మొత్తం మార్కులు 140 అయిన, Physics లో T యొక్క కొత్త శాతం (అటుఇటుగా) ఎంత ?

- (1)57 (2)32 (3)38 (4)48 (5)41

3. Fill in the blank space in order to make the sentence correct as per the given information. Total marks obtained by T in both the subjects together is more than the marks obtained by _____.

ఇచ్చిన సమాచారం ప్రతిగా, ఈ క్రింది వాక్యమును సరియైన ఖళితో పూరింపుము. T కు రెండు సబ్జెక్టులలో కలిపి వచ్చిన మార్కుల కంటే _____ మార్కులు ఎక్కువ.

- (1) Q in chemistry (2) R in physics (3) S in chemistry (4) P in physics (5) R in both the subjects together

4. What is the respective ratio between the total marks obtained by P in physics & chemistry together to the total marks obtained by T in physics & chemistry together ?

P కు physics మరియు chemistry లో వచ్చిన మార్కుల మొత్తానికి మరియు T కు physics మరియు chemistry లో వచ్చిన మార్కుల మొత్తానికి మధ్య నిష్పత్తి

- (1) 3:2 (2) 4:3 (3) 5:3 (4) 2:1 (5) None of these

5. What is the respective ratio between the total marks obtained by Q & S together in chemistry to the total marks obtained by P & R together in physics?

Q మరియు S కు chemistry లో వచ్చిన మార్కుల మొత్తానికి మరియు P మరియు R కు physics లో వచ్చిన మార్కుల మొత్తానికి మధ్య నిష్పత్తి ?

(1)23:25 (2)23:21 (3)17:19 (4)17:23 (5) none of these

21. The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, then find the other.

రెండు సంఖ్యల గ.సా.భా. 11 మరియు క.సా.గు. 693. ఒక సంఖ్య 77 అయితే, రెండవ సంఖ్యను కనుగొనుము.

22. A clock is set right at 8 A.M. The clock gains 10 minutes in 24 hours will be the true time when the clock indicates 1 P.M. on the following day?

ఒక గడియారం 8 A.M. కు సరిచేయబడింది. అది 24 గంటలలో 10 నిమిషాలు ఎక్కువగా చూపించును. అయితే మరుసటి రోజు గడియారం 1 P.M. చూపించినప్పుడు ఖచ్చిత సమయం ఎంత?

23. If $\frac{1}{5} : \frac{1}{x} :: \frac{1}{x} : \frac{1}{125}$, then the value of x is ?

$\frac{1}{5} : \frac{1}{x} :: \frac{1}{x} : \frac{1}{125}$, అయిన x విలువ ఎంత ?

24. How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/ hr ?

ఆదిత్య km/ hr వేగంతో పరుగెత్తిన, 400 m దూరం ప్రయాణించుటకు ఎన్ని నిమిషములు తీసుకొనెను?

25. Sanjay and Raju started a business and invested Rs. 20000 and Rs. 25000 respectively. After 4 months Raju left and Naresh joined by investing Rs.15000. At the end of the year there was a profit of Rs. 4600. what is the share of Naresh ?

Sanjay మరియు Raju వరుసగా Rs. 20000 మరియు Rs. 25000 పెట్టుబడితో ఒక వ్యాపారం ప్రారంభించెను. 4 నెలల తర్వాత Raju వ్యాపారం నుండి తప్పుకొనెను మరియు Naresh Rs. 15000 లతో వ్యాపారం లో చేరెను. సంవత్సరం చివరన Rs. 4600 లాభం వచ్చిన, అందులో Naresh వాట ఎంత ?

26. Meena purchased two fans each at Rs.1200. She sold one fan at the loss of 5% and other at the gain 10%. Find the total gain or loss percent ?

ఒక్కొక్కటి Rs. 1200 చొప్పున Meena రెండు fan లు కొనెను. ఒక fan ను 5% నష్టం తో, 2 వ దాన్ని 10% లాభంతో అమ్మెను. అయిన లాభం లేక నష్ట శాతం ఎంత?

P.R.GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUESTION BANK FOR ANALYTICAL SKILLS

UNIT-1 DATA ANALYSIS

1. DIRECTIONS: Study the table carefully to answer the questions that follow: Maximum and minimum Temperature (in degree Celsius) recorded on first day of each month for five different cities.

Month	Temperature									
	Bhuj		Sydney		Ontario		Kabul		Beijing	
	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min
1 st sep	24	14	12	2	5	1	34	23	12	9
1 st oct	35	21	5	-1	15	6	37	30	9	3
1 st nov	19	8	11	3	4	0	45	36	15	1
1 st dec	9	2	-5	-9	-11	-7	31	23	2	-3
1 st jan	-4	-7	-11	-13	-14	-19	20	11	5	-13

1. What is the difference between the max temperature of Ontario on 1stnov and the min temperature of Bhuj on 1stjan?

- (1) 3 °C (2) 18 °C (3) 15 °C (4) 9 °C (5) 11 °C

ANS: (5) Required difference = $4 - (-7) = 4 + 7 = 11$

2: In which month respectively the max temperature of Kabul is 2nd highest and min temperature of Sydney is highest?

- (1) 1stoct & 1stjan (2) 1stoct & 1stnov (3) 1stdec & 1stjan (4) 1stsept & 1stjan (5) 1stdec & 1stSept

ANS: (1)

3: In which month on 1st day is the difference between the max temperature & min temperature of Bhuj second highest?

- (1) 1stsept (2) 1stoct (3) 1stnov (4) 1stdec (5) 1stjan

ANS: (3) Temperature difference of Bhuj: 1st Sept: $24-14=10$ °C, 1st Nov: $19-8=11$ °C,
 1st Oct: $35-21=14$ °C, 1st Dec: $9-2=7$ °C, 1st Jan $-4+7=3$ °C

4: What is the average maximum temperature of Beijing over all the months together.

- (1) 8.4 °C (2) 9.6 °C (3) 7.6 °C (4) 9.2 °C (5) 8.6 °C

ANS: (5) Max temperature = $\frac{12+9+15+2+5}{5} = \frac{43}{5} = 8.6$ °C

5: What is the respective ratio between the min temperature of Beijing on 1stsept & the max temperature of Ontario on 1st Oct ?

- (1) 3:4 (2) 3:5 (3) 4:5 (4) 1:5 (5) 1:4

ANS: (2) required ratio = $9:15 = 3:5$

2. Study the following table carefully answer the questions percentage of marks obtained by 6 students in 6 different subjects.

Sub/student	History (out of 50)	Geography (out of 50)	Maths(out 150)	Science(out 100)	English (out of 75)	Hindi (out of 75)
Amit	76	85	69	73	64	88
Bharat	84	80	85	78	73	92
Umesh	82	67	92	87	69	76
Nikhil	73	72	78	69	58	83
Pratiksha	68	79	64	91	66	65
Ritesh	79	87	88	93	82	72

1. What is the approximately the integral % of marks obtained by umesh in all the subjects?
 (1) 81% (2) 84% (3) 86% (4) 78% (5) 77%

ANS: (1) Total marks obtained by Umesh = $41+33.5+92/100*150+87+69/100*75+76/100*5$
 $= 41+33.5+138+87+51.75+57=408.25$

Required % = $408/500*100=80\%$

2. What is the avg % of marks obtained by all the students in hindi (approximated to two places of decimal)
 (1) 77.45% (2) 79.33% (3) 75.52% (4) 73.52% (5) None of these

ANS: (2) Required avg of % in hindi = $(88+92+76+83+65+72)/6=476/6=79.33\%$

3. What is the average marks of all the students in Mathematics?
 (1) 128 (2) 112 (3) 119 (4) 138 (5) 144

ANS: (3) avg. mark in mathematics = 119

$(69+85+92+78+64+88)/6 = 150*476/100*6=119$

4. What is the average marks obtained by all the students in geography?
 (1) 38.26 (2) 37.26 (3) 37.16 (4) 39.16 (5) None of these

ANS: (4) Average marks in geography = $50(85+80+67+72+79+87)/6*1/100$
 $= 50*470/6*1/100 = 39.16$

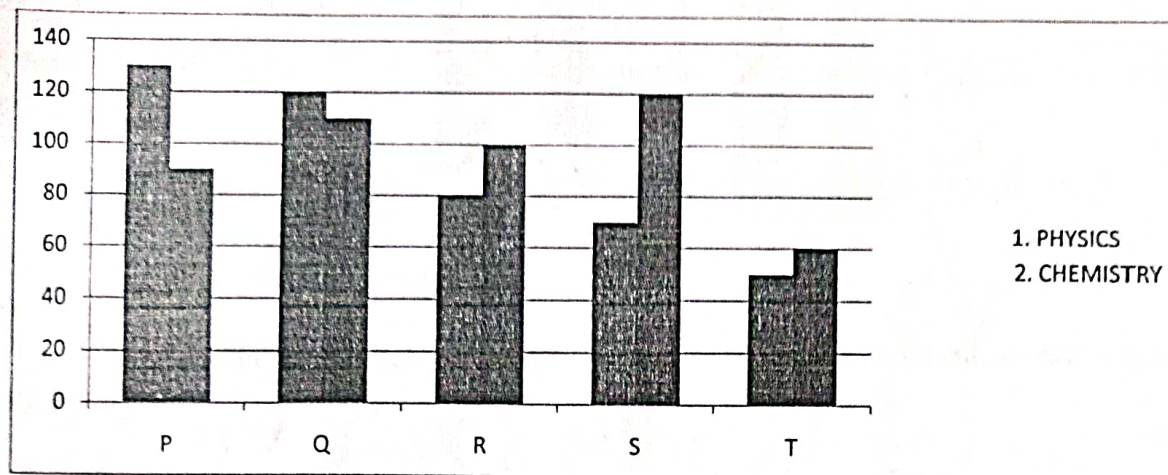
5. What are the total marks obtained by pratiksha in all the subjects taken together?
 (1) 401.75 (2) 410.75 (3) 402.75 (4) 420.75 (5) None of these

ANS: (5) marks obtained by

Ritesh = $50*68/100+50*79/100+150*64/100+91+75*66/100+50*86/100+75*65/100$
 $= 34+39.5+96+91+49.5+48.75 = 358.75$

BAR GRAPHS

1. Study the following bar graphs carefully to answer these questions. Marks obtained by 5 students in physics & chemistry.



1. Marks obtained by S in chemistry is what percent of the total marks obtained by all the students in chemistry?

(1) 25 (2) 28.5 (3) 35 (4) 31.5 (5) 22

ANS: (1) required % = $\frac{120}{90+110+100+120+60} \times 100 = \frac{120}{480} \times 100 = 25\%$

2. If the marks obtained by T in physics were increased by 14% of the original marks, what would be his new approximate % in physics if the max marks in physics were 140?

(1) 57 (2) 32 (3) 38 (4) 48 (5) 41

ANS: (5) increase in marks in physics of T = $50 \times 1.14 = 57$. Required % = $\frac{57}{140} \times 100 = 40.7 = 41$

3. What is the respective ratio between the total obtained by P in physics & chemistry together to the total marks obtained by T in physics & chemistry together?

(1) 3:2 (2) 4:3 (3) 5:3 (4) 2:1 (5) None of these

ANS: (4) required ratio = $\frac{130+90}{60+50} = \frac{220}{110} = 2:1$

4. What is the respective ratio between the total marks obtained by Q & S together in chemistry to the total marks obtained by P & R together in physics?

(1) 23:25 (2) 23:21 (3) 17:19 (4) 17:2 (5) None of these

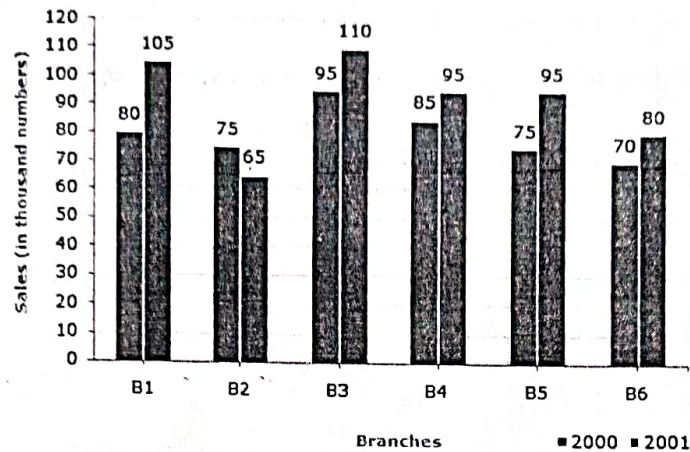
ANS: (2) Marks obtained by Q & S in chemistry = $110+120=230$.

Marks obtained by P & R in physics = $130+80=210$.

Required ratio = $\frac{230}{210} = 23:21$.

2. The bar graph given below shows the sales of books (in thousand number) from six branches of a publishing company during two consecutive years 2000 and 2001.

Sales of Books (in thousand numbers) from Six Branches - B1, B2, B3, B4, B5 and B6 of a publishing Company in 2000 and 2001.



1. What is the ratio of the total sales of branch B2 for both years to the total sales of branch B4 for both years?
- A. 2:3 B. 3:5 C. 4:5 D. 7:9

Ans: D

Explanation: Required ratio = $\frac{(75 + 65)}{(85 + 95)} = \frac{140}{180} = \frac{7}{9} = 7:9$

2. Total sales of branch B6 for both the years is what percent of the total sales of branches B3 for both the years?
- A. 68.54% B. 71.11% C. 73.17% D. 75.55%

Ans: 73.17%

Explanation: Required percentage = $\frac{70+80}{95+110} \times 100 = \frac{150}{205} \times 100 = 73.17$

3. What percent of the average sales of branches B1, B2 and B3 in 2001 is the average sales of branches B1, B3 and B6 in 2000?
- A. 75% B. 77.5% C. 82.5% D. 87.5%

Ans: 87.5%

Explanation:

Average sales (in thousand number) of branches B1, B3 and B6 in 2000 = $\frac{1}{3} \times (80 + 95 + 70) = \frac{245}{3}$

Average sales (in thousand number) of branches B1, B2 and B3 in 2001 = $\frac{1}{3} \times (105 + 65 + 110) = \frac{280}{3}$

Therefore, Required percentage = $\frac{\frac{245}{3}}{\frac{280}{3}} \times 100 = \frac{245}{280} \times 100 = 87.5$

4. What is the average sales of all the branches (in thousand numbers) for the year 2000?
- A. 73 B. 80 C. 83 D. 88

Ans. 80

Explanation: Average sales of all the six branches (in thousand numbers) for the year 2000 =

$$\frac{1}{6} \times (80 + 75 + 95 + 85 + 75 + 70) = 80$$

5. Total sales of branches B1, B3 and B5 together for both the years (in thousand numbers) is ?

A. 250

B. 310

C. 435

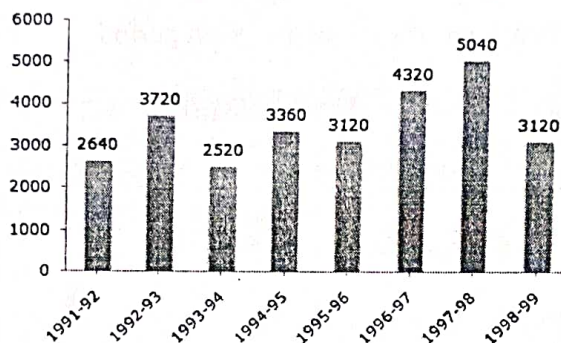
D. 560

Explanation: Total sales of branches B1, B3 and B5 for both the years (in thousand numbers)

$$= (80 + 105) + (95 + 110) + (75 + 95) = 560.$$

3. The bar graph given below shows the foreign exchange reserves of a country (in million US \$) from 1991 - 1992 to 1998 - 1999.

Foreign Exchange Reserves of a Country. (in million US \$)



The ratio of the number of years, in which the foreign exchange reserves are above the average reserves, to those in which the reserves are below the average reserves is?

A. 2:6

B. 3:4

C. 3:5

D. 4:4

Ans: 3:5

Explanation: Average foreign exchange reserves over the given period = 3480 million US \$.

The country had reserves above 3480 million US \$ during the years 1992-93, 1996-97 and 1997-98, i.e., for 3 years and below 3480 million US \$ during the years 1991-92, 1993-94, 1994-95, 1995-56 and 1998-99 i.e., for 5 years.

Hence, required ratio = 3:5.

The foreign exchange reserves in 1997 - 98 was how many times that in 1994-95?

A. 0.7

B. 1.2

C. 1.4

D. 1.5

Ans: 1.5

Explanation: Required ratio = $\frac{5040}{3360} = 1.5$

For which year, the percent increase of foreign exchange reserves over the previous year, is the highest?

A. 1992-93

B. 1993-94

C. 1994-95

D. 1996-97

Ans: 1992-93

Explanation: There is an increase in foreign exchange reserves during the years 1992 - 1993, 1994 - 1995, 1996 - 1997, 1997 - 1998 as compared to previous year (as shown by bar-graph).

The percentage increase in reserves during these years compared to previous year are:

$$\text{For } 1992 - 1993 = \frac{3720 - 2640}{2640} \times 100 = 40.91$$

$$\text{For } 1996 - 1997 = \frac{4320 - 3120}{3120} \times 100 = 38.46$$

$$\text{For } 1997 - 1998 = \frac{5040 - 4320}{4320} \times 100 = 16.67$$

Clearly, the percentage increase over previous year is highest for 1992 - 1993.

4. The foreign exchange reserves in 1996-97 were approximately what percent of the average foreign exchange reserves over the period under review?

A. 95% B. 110% C. 115% D. 125%

Ans: 125%

Explanation: Average foreign exchange reserves over the given period

$$= \frac{1}{8} \times (2640 + 3720 + 2520 + 3360 + 3120 + 4320 + 5040 + 3120)$$

$$= 3480 \text{ million US \$}$$

Foreign exchange reserves in 1996 - 1997 = 4320 million US \$.

$$\text{Required percentage} = \frac{4320}{3480} \times 100 = 124.14 \approx 125.$$

5. What was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94?

A. 100 B. 150 C. 200 D. 620

Ans: 100%

Explanation: Foreign exchange reserves in 1997 - 1998 = 5040 million US \$.

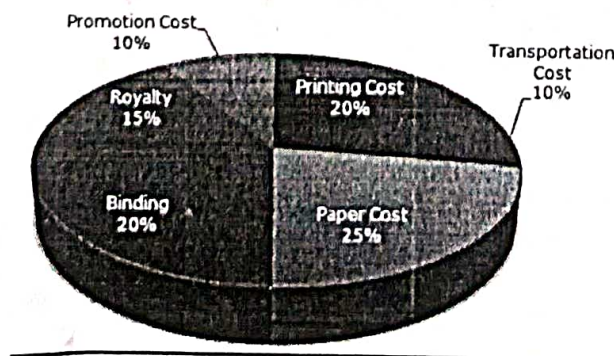
Foreign exchange reserves in 1993 - 1994 = 2520 million US \$.

$$\text{Increase} = 5040 - 2520 = 2520 \text{ US \$}.$$

$$\text{Percentage increase} = \frac{2520}{2520} \times 100 = 100$$

4. The following pie-chart shows the percentage distribution of the expenditure incurred in publishing a book. Study the pie-chart and the answer the questions based on it.

Expenditures (in percentage) Incurred in Publishing a Book



1. If for a certain quantity of books, the publisher has to pay Rs. 30, 600 as printing cost, then what will be amount of royalty to be paid for these books?

- A. Rs.19,450 B. Rs.21,200 C. Rs.22,950 D. Rs.26,150

Ans: Rs.22,950

Explanation: Let the amount of Royalty to be paid for these books be Rs. r .

Then, $20 : 15 = 30600 : r \Rightarrow \frac{30600 \times 15}{20} = \text{Rs. } 22,950$

2. What is the central angle of the sector corresponding to the expenditure incurred on Royalty?

- A. 15° B. 24° C. 54° D. 48°

Ans: 54°

Explanation: Central angle corresponding to Royalty = $(15\% \text{ of } 360)^\circ = \left(\frac{15}{100} \times 360\right)^\circ = 54^\circ$

3. The price of the book is marked 20% above the C.P. If the marked price of the book is Rs. 180, then what is the cost of the paper used in a single copy of the book?

- A. Rs.36 B. Rs.37.50 C. Rs.42 D. Rs.44.25

Explanation:

Clearly, marked price of the book = 120% of C.P.

Also, cost of paper = 25% of C.P

Let the cost of paper for a single book be Rs. n .

Then, $120 : 25 = 180 : n \Rightarrow n = \frac{25 \times 180}{120} = \text{Rs. } 37.50$

4. If 5500 copies are published and the transportation cost on them amounts to Rs. 82500, then what should be the selling price of the book so that the publisher can earn a profit of 25%?

- A. Rs.187.50 B. Rs.191.50 C. Rs.175 D. Rs.180

Ans: Rs.187.50

Explanation:

For the publisher to earn a profit of 25%, S.P. = 125% of C.P.

Also Transportation Cost = 10% of C.P.

Let the S.P. of 5500 books be Rs. x .

Then, $10 : 125 = 82500 : x \Rightarrow x = \frac{125 \times 82500}{10} = 1031250, \text{ Rs}$

S.P. of one book = $\text{Rs. } \frac{1031250}{5500} = 187.50$

5. Royalty on the book is less than the printing cost by:

A. 5%

B. 33 1/5 %

C. 20%

D. 25%

Ans: 25%

Explanation:

Printing Cost of book = 20% of C.P.

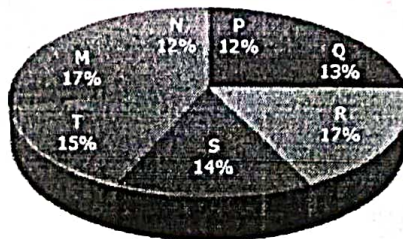
Royalty on book = 15% of C.P.

Difference = (20% of C.P.) - (15% of C.P.) = 5% of C.P.

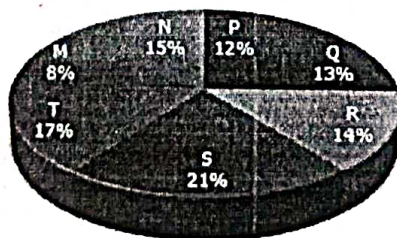
Percentage difference = $\frac{\text{difference}}{\text{printing cost}} \times 100 = \frac{5\% \text{ of C.P.}}{\text{Printing Cost}} \times 100 = 25\%$

3. The following pie-charts show the distribution of students of graduate and post-graduate levels in seven different institutes in a town.

Total Number of Students of Graduate Level = 27300



Total Number of Students of Post-Graduate Level = 24700



Distribution of students at graduate and post-graduate levels in seven institutes

1. What is the total number of graduate and post-graduate level students in institute R?

A. 8320

B. 7916

C. 9116

D. 8099

Ans: 8099

Explanation: Required number = (17% of 27300) + (14% of 24700) = 4641 + 3458 = 8099.

2. What is the ratio between the number of students studying at post-graduate and graduate levels respectively from institute S?

A. 14:19

B. 19:21

C. 17:21

D. 19:14

Ans: 19:14

Explanation: Required ratio = $\frac{(21\% \text{ of } 24700)}{(14\% \text{ of } 27300)} = \frac{(21 \times 24700)}{14 \times 27300} = \frac{19}{14}$

3. How many students of institutes of M and S are studying at graduate level?

- A. 7516 B. 8463 C. 9127 D. 9404

Explanation: Students of institute M at graduate level = 17% of 27300 = 4641.
 Students of institute S at graduate level = 14% of 27300 = 3822.

Total number of students at graduate in institutes M and S = (4641 + 3822) = 8463

4. What is the ratio between the number of students studying at post-graduate level from institutes S and the number of students studying at graduate level from institute Q?

- A. 13:19 B. 21:13 C. 13:8 D. 19:13

Explanation: Required ratio = $\frac{21\% \text{ of } 24700}{13\% \text{ of } 27300} = \frac{21 \times 24700}{13 \times 27300} = \frac{19}{13}$

5. Total number of students studying at post-graduate level from institutes N and P is

- A. 5601 B. 5944 C. 6669 D. 8372

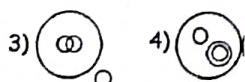
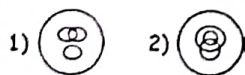
Explanation: Required number = (15% of 24700) + (12% of 24700) = 3705 + 2964 = 6669

VENN DIAGRAM

1. Which of the following Venn- diagram correctly illustrates the relationship among the classes : Tennis fans, Cricket players, Students



2. In a dinner party both fish and meat were served. Some took only fish and Some only meat. There were some vegetarians who did not accept either. The rest accepted both fish and meat. Which of the following Venn-diagrams correctly reflects this situation?



UNIT-2

I In each of the following questions, a number series is given with one term missing. Choose the correct alternative that will continue the same pattern and replace the question mark in the given series.

1. 1,9,25,49,?,121

a)64 b)81 c)91 d)100 ()

2. 11,13,17,19,23,25,?

a)26 b)27 c)29 d)37 ()

3. 6,11,21,36,56,?

a)42 b)51 c)81 d)91 ()

4. 10,18,28,40,54,70,?

a)85 b)86 c)87 d)88 ()

5. 22,24,28,?,52,84

a)36 b)38 c)42 d)46 ()

6. 28,33,31,36,?,39

a)32 b)34 c)38 d)40 ()

7. 6,17,39,72,?

a)83 b)94 c)116 d)127 ()

8. 325,259,204,160,127,105,?

a)94 b)96 c)98 d)100 ()

II In each of the following questions, one term in the number series is wrong. Find out the wrong term

1. 3,10,27,4,16,64,5,25,105

a)3 b)4 c)10 d)27 ()

2. 8,13,21,32,47,63,83

a)13 b)21 c)32 d)47 ()

3. 105,85,60,30,0,-45,-90

a)105 b)60 c)0 d)-45 ()

4. 325,259,202,160,127,105,94

a)94 b)127 c)202 d)259 ()

5. 1,2,4,8,16,32,64,96

a)4 b)32 c)64 d)96 ()

6. 10,26,74,218,654,1946,5834

a)26 b)74 c)218 d)654 ()

7. 1,3,10,21,64,129,356,777

a)21 b)129 c)10 d)356 ()

8. 3,4,10,32,136,685,4116

a)10 b)32 c)136 d)4116 ()

III In each of the following questions, various terms of an alphabet series are given with one or more terms missing as shown by (?). Choose the missing terms out of the given alternatives.

1. R, U, X, A, D, ?

a)F b)G c)H d)I ()

2. T, R, P, N, L, ?, ?

a)J,G b)J,H c)K,H d)K,I ()

3. a,b,c,f,?,h, g,?,i

a)e,j b)e,k c)f,j d)j,e ()

4. Z,Y,X,U,T,S,P,O,N,K,?,?

a)G,H b)H,I c)I,H d)J,I ()

5. A,B,N,C,D,O,E,F,P,?,?,?

a)G,H,I b)G,H,J c)G,H,Q, d)J,K,L ()

6. A,B,B,D,C,F,D,H,E,?,?

a)E,F b)F,G c)F,I d)J,F e)j,k ()

7. C,Z,F,X,I,V,L,T,O,?,?

a)O,P b)P,Q c)R,R d)S,R ()

8. AB,DEF,HIJK,?,STUVWX

- a)LMNO b)LMNOP c)MNOPQ d)QRSTU ()

IV In each of the following questions, a letter-number series is given with one or more terms missing as shown by (?). Choose the missing term out of the given alternatives.

1. D-4, F-6, H-8, J-10, ?, ?

- a) K -12, M-13 b)L-12,M-14 c)L-12,N-14 d)K-12,M-14 ()

2. 3F,6G, 11I, 18L, ?

- a)21O b)25N c)25P d)27P e) 27Q ()

3. W-144, ?, S-100, Q-81, O-64

- a)U-121 b) U-122 c)V-121 d)V-128 ()

4. 2Z5, 7Y7, 14X9, 23W11, 34V13, ?

- a)27U24 b)45U15 c)47U15 d)47V14 ()

5. N5V, K7T, ? ,E14P, B19N

- a)H9R b)H10Q c)H10R d)I10R ()

6. find the term which does not fit into the series

1CV,5FU,9IT,15LS,17OR

- a)5FU b)15LS c)9IT d)17OR ()

7. Q1F, S2E, U6D, W21C, ?

- a)Y44B b)Y66B c)Y88B d)Z88B ()

V In each of the following letter series, some of the letters are missing which are given in order as one of the alternatives below it. Choose the correct alternative.

1. __ aba __ ba _ ab

- a)abbba b)abbab c)baabb d)bbaba ()

2. ab __ baa __ ab _

- a)aaaaa b)aabaa c)aabab d)baabb ()

3. a _ ba _ b _ b _ a _ b

- a)abaab b)abbab c)aabba d)bbabb ()

4. _ op _ mo _ n _ _ pnmop _

- a) mnpmon b) mpnnop c) mnompn d) mnpomn ()

5. _ nmmn _ mmmn _ mnnm _

- a) nmmn b) mnnm c) nnmm d) nmnm ()

6. ba _ cb _ b _ bab _

- a) acbb b) back c) bcaa d) cabb ()

7. _ aa _ ba _ bb _ ab _ aab

- a) aaabb b) babab c) bbaab d) bbaa ()

8. ab _ d _ aaba _ na _ badna _ b

- a) andaa b) babda c) badna d) dbanb ()

VI Find out the relationship between the first two words and choose the word from the given alternatives.

1. Anaemia : Blood :: Anarchy : ?

- a) Lawlessness b) Government c) Monarchy d) Disorder ()

2. Botany : Plants :: Entomology : ?

- a) Snakes b) Insects c) Birds d) Germs ()

3. Menu : Food :: Catalogue :

- a) Rack b) Newspaper c) Library d) Books ()

4. Pulp : Paper :: Hemp : ?

- a) Basket b) Yarn c) Rope d) Cotton ()

5. Moon : Satellite :: Earth : ?

- a) Sun b) Planet c) Solar System d) Asteroid ()

6. Coconut : Shell :: Letter : ?

- a) Letter – Box b) Stamp c) Mail d) Envelope ()

7. Assam : Bihu :: Kerala : ?

- a) Kathakali b) Kuchipudi c) Kathak d) Bharatanatyam ()

8. Man : Machine :: Master : ?

- a) Worker b) Manager c) House d) Slave ()

VII

1. Necklace is related to Jewellery in the same way as Shirt is related to ?

- a) Thread b) Cloth c) Cotton d) Apparel ()

2. Needle is related to Thread in the same way as Pen is related to ?

- a) Ink b) Cap c) Paper d) Word e) Stationery ()

3. Drama is related to Director in the same way as Magazine is related to ?

- a) Story b) Editor c) Reader d) Printer ()

4. Wax is related to Grease in the same way as Milk is related to ?

- a) Drink b) Ghee c) Curd d) Protein ()

5. Impossible is related to Feasible in the same way as Theoretical is related to ?

- a) Radical b) Usable c) Practical d) Workable ()

6. Cyclone is related to Anticyclone in the same way as Flood is related to ?

- a) Devastation b) Havoc c) River d) Drought ()

7. Earth is related to Axis in the same way as Wheel is related to ?

- a) Tyre b) Car c) Road d) Hub ()

8. Income is related to Profit in the same way as Expenditure is related to ?

- a) Balance b) Loss c) Sale d) Receipt e) Surplus ()

VII Find out the relationship between the first two numbers and choose the number from the given alternatives.

1. 18 : 30 :: 36 : ?

- a) 54 b) 62 c) 64 d) 66 ()

2. 6 : 222 :: 7 : ?

- a) 210 b) 336 c) 343 d) 350 ()

3. 14 : 9 :: 26 : ?

- a) 12 b) 13 c) 15 d) 31 ()

4. 8 : 28 :: 27 : ?

- | | | | | |
|---------------------------|--------|--------|--------|-----|
| a)55 | b)63 | c)64 | d)65 | () |
| 5. $68 : 130 :: ? : 350$ | | | | |
| a) 210 | b)216 | c)222 | d)240 | () |
| 6. $42 : 56 :: 72 : ?$ | | | | |
| a)81 | b)90 | c)92 | d)100 | () |
| 7. $9 : 80 :: 100 : ?$ | | | | |
| a)901 | b)1009 | c)9889 | d)9999 | () |
| 8. $149 : 238 :: 159 : ?$ | | | | |
| a) 169 | b) 248 | c)261 | d)268 | () |

UNIT-3

BODMASRULE AND SIMPLIFICATION

- $12573+43495+23472=?$
- $(8 \div 88) \times 8888088=?$
- The value of $1001 \div 11$ of 13 is?
- $20\frac{1}{2} + 30\frac{1}{3} - 15\frac{1}{6}=?$
- Simplify $2-[2 - \{2 - 2(2 + 2)\}]=?$
- Simplify $18-[5 - \{6 + 2(7 - 8 - 5)\}]$.
- $(-5)(4)(2)(-\frac{1}{2})(\frac{3}{4})=?$
- Find the value of $\frac{(6+6+6+6)+6}{4+4+4+4+4}$
- What is the value of $\frac{(P+Q)}{(P-Q)}$ if $\frac{P}{Q}=7$?

DECIMAL FRACTIONS

- If $204 \div 12.75 = 16$, then $2.04 \div 1.275 = ?$ *1-6*
- $0.03 \times 0.0124 = ?$ *0.000372*
- $7212 + 15.231 - ? = 6879$ *348.231*
- $4211.01 + 22.261 - ? = 2645.759$ *1587.512*
- $0.004 \times 0.5 = ?$ *0.002*
- $24.39 + 562.093 + 35.96 = ?$ *622.443*
- $926 + 9.026 + 0.926 + 9.0026 = ?$ *944.9546* $(a+b)^2$
- The expression $(12.86 \times 12.86 + 12.86 \times p + 0.14 \times 0.14)$ will be a perfect square for p equal to *0.28*
 $a^2 + 2ab + b^2$
 $p = 2b$
 $p = 2 \times 0.14 = 0.28$

DIVISIBILITY RULE

1. If the the number $5*2$ is divisible by 6 then *?
2. If the number $517*324$ is completely divisible by 3 , then the smallest whole number in Place of * will be.
3. If the number $481*673$ is completely divisible by 9, then the smallest whole number in Place of * will be.
4. If the number $97215*6$ is completely divisible by 11, then the smallest whole number in Place of * will be.
5. If the number $91876*2$ is completely divisible by 8, then the smallest whole number in Place of * will be.
7. Find the least value of * for which $7*5462$ is divisible by 9
8. Find the least value of * for which $4832*18$ is divisible by 11.

LCM and HCF:

1. What is the lowest common multiple of 12, 36 and 20? 180
2. Find the H.C.F of 108,288and 360. 36
3. Find the greatest common divisor of 24 and 16 8
4. Two numbers are in the ratio 2 : 3. If their L.C.M. is 48. what is sum of the numbers? 40
5. The ratio of two numbers is 4 : 5. If the HCF of these numbers is 6, what is their LCM? 120
6. The H.C.F. of two numbers is 5 and their L.C.M. is 150. If one of the numbers is 25, then the other is: 30
7. The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, then find the other. 99
8. Find L.C.M of $\frac{28}{3}$ $\frac{16}{9}$ and $\frac{10}{27}$ $\frac{40}{3}$

BLOOD REALTIONS

1. A is B's sister. C is B's mother. D is C's father. E is D's mother. Then, how is A related to D?
2. Pointing out to a lady, a girl said, "She is the daughter-in-law of the grandmother of father's only son." How is the lady related to the girl ?
3. There are six persons A. B, C, D, E and F. C is the sister of F. B is the brother of husband. D is the father of A and grandfather of F. There are two fathers, three brothers ; a mother in the group. Who is the mother ?

4. Pointing to a person, a man said to a woman, "His mother is the only daughter of your father. How was the woman related to the person ?
5. A girl introduced a boy as the son of the daughter of the father of her uncle. What is the relation between the boy and the girl ?
6. In a family, there are six members A, B, C, D, E and F. A and B are a married couple, A being the male member. D is the only son of C, who is the brother of A. E is the sister of D. B is the daughter-in-law of F, whose husband has died. How is E related to C ?
7. A woman introduces a man as the son of the brother of her mother. How is the man, related to woman ?

CALENDAR

1. What was the day on 15th August 1947 ?
2. Today is Monday. After 61 days, it will be?
3. The last day of a century cannot be?
4. What was the day of the week on, 16th July, 1776?
5. It was Sunday on Jan 1, 2006. What was the day of the week Jan 1, 2010?
6. What was the day of the week on 28th May, 2006?
7. What will be the day of the week 15th August, 2010?
8. If 6th March, 2005 is Monday, what was the day of the week on 6th March, 2004?

CLOCKS

1. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours will be the true time when the clock indicates 1 p.m. on the following day?
2. At what time between 4 and 5 o'clock will the hands of a watch point in opposite directions?
3. An accurate clock shows 8 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon?
4. A clock is set right at 5 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?
5. At what time between 5 and 6 o'clock are the hands of a clock 3 minutes apart ?
6. Find the angle between the hour hand and the minute hand of a clock when the time is 3.25?
7. At what angle the hands of a clock are inclined at 15 minutes past 5?
8. At what time between 2 and 3 o'clock will the hands of a clock be together?

$$a:b::c:d$$

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{1}{5} = \frac{1}{x}$$

$$\frac{1}{x} = \frac{B}{4}$$

$$A = \frac{125}{3}$$

$$a:b = \frac{20}{10.5} = \frac{125}{21}$$

$$b:c = \frac{125}{21}$$

$$a:c = 625$$

$$x = 25$$

Unit -IV

AVERAGE:

1. The average of 5, 10,15,20,25?
2. Find the average of first 40 natural numbers.
3. The average of four consecutive even numbers is 27. Find the largest of these numbers.
4. The average of four consecutive odd numbers is 61 what is the difference between the highest and lowest numbers?
5. The average of 5 numbers is 15 and the average of first three numbers is 10. What is the average of last two numbers?
6. The average age of 15 students of a class is 15 years. Out of these, the average age of 5 student is 14 year and that of the other 9students is 16 years. The age of the 15th student is?
7. The average of 5 numbers is 15 and the average of first three numbers is 10 and the average last three numbers is 20. Then find the middle number?
8. The average of five numbers is 27. If one number is excluded, the average becomes 25. The excluded number is:

RATION & PROPORTION:

1. If $A : B = 2 : 3$ $B : C = 4 : 7$ then find $A : B : C = ?$
2. If $A : B = 2 : 3$ $B : C = 3 : 4$ then find $A : B : C = ?$
3. If $a:b=2:3$ and $b:c=3:5$ then find $a:c=?$
4. If $2A=3B$ and $4B=5C$, then $A:C$ is /
5. Find the mean proportional of 9 and 25
6. Find the third proportional to 16 and 4
7. If $\frac{A}{3} = \frac{B}{4} = \frac{C}{5}$, then $A:B:C$ is /
8. If $\frac{1}{5} : \frac{1}{x} :: \frac{1}{x} : \frac{1}{125}$, then the value of x is /

$$\sqrt{9 \times 25} = \sqrt{225}$$

$$15 \quad 8 : 12 : 21$$

$$\frac{A}{B} = \frac{3}{2} \quad \frac{B}{C} = \frac{5}{4}$$

$$\frac{A}{B} \times \frac{B}{C} = \frac{3}{2} \times \frac{5}{4}$$

$$\frac{A}{C} = \frac{15}{8}$$

$$A : C = 15 : 8$$

$$\frac{a}{b} = \frac{2}{3} \quad \frac{b}{c} = \frac{3}{5}$$

$$\frac{a}{b} \times \frac{b}{c} = \frac{2}{3} \times \frac{3}{5}$$

$$\frac{a}{c} = \frac{2}{5}$$

$$\frac{B^2}{AB} = \frac{16}{16}$$

PROBLEM ON AGES:

1. A father said his son , " I was as old as you are at present at the time of your birth. " If the father age is 38 now, the son age 5 years back was :
2. The total age of A and B is 12 years more than the total age of B and C. C is how many years younger than A ?
3. In 10 years, A will be twice as old as B was 10 years ago. If A is now 9 years older than B, the present age of B is :
4. The age of a man is 4 times of his son. Five years ago, the man was nine times old as his son was that time. The present age of man is?
5. The sum of the present ages of a father and his son is 60 years. five years ago, father's age was 1 times the age of the son. so now the son's age will be:
6. Six years ago Anita was P times as old as Ben was. If Anita is now 17 years old, how old is Ben now in terms of P ?

- Sachin is younger than Rahul by 7 years. If the ratio of their ages is 7:9, find the age of Sachin.
- The ratio of the present ages of P and Q is 3 : 4. Five years ago, the ratio of their ages was 5 : 7. Find their present ages.

TIMES AND DISTANCE-SPEED

- An athlete runs 200 metres race in 24 seconds. His speed is?
- How many minutes does Aditya take to cover a distance of 400m, if he runs at a speed of 20 km/hr?
- A car is running at speed of 108kmph. What distance will it cover in 15 seconds?
- A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?
- Peter can cover a certain distance in 1hr.24min. by covering two third of the distance at 4km and the rest at 5kmph. Find the total distance.
- A and B are two stations 390km apart. A train starts from A at 10 a.m. and travels towards B at 65kmph. Another train starts from B at 11 a.m. and travels towards A at 35 kmph. At what time do they meet?

UNIT-V

PERCENTAGES

- $8\frac{1}{3}\%$ expressed as fraction is ?
- 2 is what percent of 50?
- What percent of $\frac{1}{2}$ is $\frac{1}{3}$?
- X% of Y is Y% of, ?
- What is 25% of 25% equal to?
- 30% of 140 = ?, % of 840
- 5 % of (50% of Rs 300) is?
- 270 candidates appeared in an examination, of which 252 passed. The pass percentage is.

PROFIT AND LOSS

- A man buys a cycle for Rs.1400 and sells it at a loss of 15%. What is the selling price of the cycle?
- The CP of 21 articles is equal to SP of 18 articles. Find the gain (or) loss percent
- A man buys an article for Rs 27.50 and sells it for Rs 28.60. find his gain percent ?
- An article is bought for RS. 450 and sold for Rs.400 .what is the loss%?
- When a commodity is sold for Rs. 34,80 there is a loss of 25%, what is the cost price of commodity?
- An article is sold at certain price. By selling it at $\frac{2}{3}$ of that price one loses 10%. Find the gain percent at original price..
- Meena purchased two fans each at Rs.1200. She sold one fan at the loss of 5% and other at the gain 10%. Find the total gain or loss percent?

PARTNERSHIP

1. Dhilip and Manohar started a business by investing Rs.100000 and Rs.150000 respectively. Find the share of each out of a profit of Rs.24000?
2. Sanjay and Raju started a business and invested Rs.20000 and Rs.25000 respectively. After 4 months Raju left and Naresh joined by investing Rs.15000. At the end of the year there was a profit of Rs.4600. What is the share of Naresh?
3. Three partners A, B, C start a business. Twice the investment of A is equal to thrice the capital of B and the capital of B is four times the capital of C. Find the share of each out of a profit of Rs.297000?
4. A, B, C hire meadow for Rs.2934.60. A puts in 10 oxen for 20 days; B 30 oxen for 8 days and C 16 oxen for 9 days. Find the rent paid by each? *1005, 1206, 723.6*
5. A and B started a business in partnership by investing Rs.8000 and Rs.7000 respectively. If at the end of a year, a profit of Rs.22,500 was earned. What is the share of A?
6. In partnership business, A has invested Rs.4200 while B has invested a certain amount. If out of the overall profit of Rs.600, A's share is Rs.320, what is the amount invested by B (in Rs)?
7. Chetan and Suman started a business in partnership by investing Rs.15000 and Rs.18000 respectively. If at the end of the year, Chetan's share in the profit was Rs.1200, what was the amount of total profit?
8. In a partnership business, A has invested 2000 for 5 months, while B has invested Rs.3500 for a certain period. If out of the total annual profit of Rs.1440, B's share has been Rs.840. For how many months has he kept his investment in the business?

SIMPLE AND COMPOUND INTEREST

1. Find the simple interest on Rs 7500 in 4 years at 15% .
2. The simple interest on Rs. 6400 at $12\frac{1}{2}$ % per annum is Rs.2000, find the period
3. On what sum of money will the simple interest be Rs.2000 in 5 years 8% per annum?
4. A sum of Rs 1600 gives a simple interest of Rs252 in 2 years and 4 months. The rate of interest per annum is ?
5. Find the compound interest on Rs 8000 for 3 years at 5% per annum
6. A sum of Rs.3000 is lent for 3 years at 10% p.a compound interest. Find the amount
7. Find the amount on Rs 7500 at 4% per annum for 2 years compounded annually.
8. Find the compound interest on Rs.15,625 for 9 months at 16% per annum compounded Quarterly.

$$\begin{array}{l} 300 \\ 300 + 20 \\ 300 + 63 \end{array}$$

$$630$$

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
III B.Sc. MATHEMATICS-Semester V (W.e.f. 2018-2019)
Course: Ring Theory & Vector Calculus

Total Hrs. of Teaching-Learning: 75 @ 5 hr/Week

Total credits: 5

OBJECTIVES:

- To impart knowledge on Ring Theory and its applications.
- To make awareness of the concepts of the transformation between line Integral, Surface Integral and Volume integral.
- To introduce the concepts of geometrical meaning of Gradient, Divergence and Curl

RING THEORY

Unit – I: Rings – I

(15 hrs)

Definition of Ring and Basic Properties, Boolean Rings, Divisors of Zero and Cancellation Laws in Rings, Integral Domain, Division Ring and Fields, The Characteristic of a Ring – The Characteristic of an Integral Domain, the Characteristic of a Field, Sub Rings and Ideals.

Unit – II: Rings – II

(15 hrs)

Definition of Homomorphism – Homomorphic Image – Elementary Properties of Homomorphism – Kernel of a Homomorphism – Fundamental Theorem of Homomorphism – Maximal Ideals – Prime Ideals.

VECTOR CALCULUS

UNIT – III: Vector Differentiation

(15 hrs)

Vector differentiation – Ordinary Derivatives of Vector valued functions, Continuity and Differentiation, Gradient, Divergence, Curl operators, Formulae involving these operators.

UNIT – IV: Vector Integration

(15 hrs)

Line Integral, Surface Integral, Volume Integrals with examples.

Unit – V: Vector Integration Applications

(15 hrs)

Gauss Divergence Theorem, Stokes theorem, Green's Theorem in plane and applications of these theorems.

Additional Inputs: Euclidean Ring definition and Examples.

Prescribed text Book:

A text book of Mathematics, Vol. III, S. Chand & Co.

Books for Reference:

1. Topics in Algebra by I.N.Herstine
2. Abstract Algebra by J. Fraleigh, Published by Narosa Publishing house
3. Vector Calculus by Santhi Narayan, Published by S.Chand & Company Pvt. Ltd., New Delhi
4. Vector Calculus by R.Gupta, Published by Laxmi Publications.

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-V, PAPER-V

Unit	TOPIC	V.S.A.Q	S.A.Q(including choice)	E.Q(including choice)	Total Marks
I	Rings – I	01	01	02	22
II	Rings – II	01	01	02	22
III	Vector differentiation	01	01	02	22
IV	Vector integration	01	01	01	14
V	Vector Integration Applications	01	01	01	14
TOTAL		05	05	08	94

E.Q = Essay questions (8 marks)
S.A.Q = Short answer questions (5 marks)
V.S.A.Q = Very Short answer questions (1 mark)

Essay questions : 5 x 8 M = 40
Short answer questions : 3 x 5 M = 15
Very Short answer questions : 5 x 1 M = 05

Total Marks _____ = 60

P.R. Government College (Autonomous), Kakinada
III year B.Sc., Degree Examinations - V Semester
Mathematics: Ring Theory & Vector Calculus
Paper – V (Model Paper w. e. f. 2019-2020)

Time: 2 Hrs 30 Min

Max. Marks: 60M

PART – I

Answer ALL the following questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Define Boolean Ring.
2. Find Kernel of the Homomorphism $f: Z(\sqrt{2}) \rightarrow Z(\sqrt{2})$ defined by $f(m + n\sqrt{2}) = m - n\sqrt{2}, \forall m + n\sqrt{2} \in Z(\sqrt{2})$.
3. Find div f, where $f = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.
4. Evaluate $\int_0^1 (e^t \bar{i} + e^{-2t} \bar{j}) dt$.
5. State the Green's Identities.

PART – II

Answer any THREE of the following questions. Each question carries 5 marks. 3 x 5 = 15 M

6. Show that a ring R has no zero divisors if and only if the cancellation laws hold in R.
7. Let R and R' be two rings and $f: R \rightarrow R'$ be a homomorphism. Then prove that the Kernel of f is an ideal of R.
8. Prove that $\text{div Curl } \bar{f} = 0$.
9. If $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$, find the circulation of F round the curve C, where C is the Circle $x^2 + y^2 = 1, z = 0$.
10. Evaluate $\oint_C (\cos x \cdot \sin y - xy) dx + \sin x \cdot \cos y dy$, by Green's theorem, where C is the circle $x^2 + y^2 = 1$.

PART – III

Answer any FIVE questions from the following by choosing at least TWO from each section.
Each question carries 8 marks. 5 X 8 = 40 M

SECTION – A

11. If $E = \{0, 1, 2, 3, 4\}$, then prove that $(E, +_5, \times_5)$ form a field. Justify your answer.
12. Define the characteristic of a ring. Prove that the characteristic of an integral domain is either a prime or zero.
13. State and Prove fundamental theorem of homomorphism in rings.
14. Show that an ideal U of a commutative ring R with unity is maximal if and only if the quotient ring R/U is a field.

SECTION - B

15. Prove that $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$.
16. Find the directional derivative of $f = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.
17. If $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$, evaluate $\int \vec{F} \cdot \vec{N} \, dS$ where S is the surface of the cube bounded by $x = 0, x = a, y = 0, y = a, z = 0, z = a$.
18. Verify Stokes theorem for $A = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

PR GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank

PAPER-V: RING THEORY AND VECTOR CALCULUS

Very Short Answer Questions

Unit-I

1. Define ring and give one example.
2. Define Boolean ring with example.
3. Write the set of zero divisors of $(Z_{12}, +_{12}, \times_{12})$.
4. Define integral domain and give one example.
5. Write the units of the ring of Gaussian Integers.
6. Give an example of a division ring which is not a field.
7. Define field and give one example.
8. What is the characteristic of $(Z_{12}, +_{12}, \times_{12})$.
9. Find all the associates of $(2 - i)$ in the ring of Gaussian integers.
10. Define subring and explain with one example.
11. Define subfield and explain with one example.
12. Define ideal and explain with one example.
13. Give an example of a subring which is not an ideal.
14. Define principal ideal and explain with one example
15. Define principal ideal ring and explain with one example
16. Define quotient ring.

Unit-II

17. Define homomorphism, homomorphic image, monomorphism, epimorphism.
18. Define isomorphism and automorphism.
19. Define kernel of homomorphism.
20. Is the ring $2Z$ isomorphic to the ring $3Z$.
21. Define Maximal ideal and Prime ideal with example.
22. Give an example of a prime ideal, which is not a maximal.

Unit-III

23. Define scalar point function and vector point function
24. Prove that $\nabla \cdot \vec{r} = \frac{\vec{r}}{r}$
25. Define divergence of a vector.
26. Define Solenoidal vector.
27. Define Laplacian operator.
28. If $\phi = x^2 - y^2$, then show that $\nabla^2 \phi = 0$.
29. Define curl of a vector
30. Define irrotational vector.
31. If $F = (x + 3y)i + (y - 2z)j + (x + pz)k$ is solenoidal, then find p ?
32. Define scalar potential of an irrotational vector

Unit-IV

33. Define definite integral
34. Find the value of \bar{r} satisfying the equation $\frac{d^2 \bar{r}}{dt^2} = \bar{a}t + \bar{b}$, where \bar{a}, \bar{b} are constant vectors.
35. Define smooth curve.
36. Define line integral.
37. Define circulation.
38. Define surface integral.
39. Define flux.
40. Define volume integral

Unit-V

41. State Gauss divergence theorem.
42. Apply Gauss's theorem to prove that $\int_s \bar{r} \cdot N ds = 3V$
43. State Green's theorem
44. State Stokes theorem
45. Prove that $\text{curl grad } \phi = 0$ by using Stokes theorem

Short Answer Questions

Unit-I

1. If R is a Boolean ring, then prove that
 - i. $a + a = 0, \forall a \in R$
 - ii. $a + b = 0 \Rightarrow a = b$
 - iii. R is commutative under multiplication
2. Prove that, a ring R has no zero divisors iff the cancellation laws hold in R .
3. Prove that every field is an integral domain
4. Prove that, the intersection of two subrings of a ring R is a subring of R
5. If R is a commutative ring and $a \in R$, then prove that $Ra = \{ra : r \in R\}$ is an ideal of R
6. Prove that, the characteristic of a Boolean ring is 2.
7. Prove that, is the set of integers a subring of set of rational numbers w.r.t. usual addition and multiplication.
8. Prove that, every subring is an ideal. Explain briefly.
9. If U is an ideal of a ring R with unity element and $1 \in U$, then $U = R$.
10. Prove that, the intersection of two ideals of a ring R is an ideal of R
11. Prove that the characteristic of an integral domain is either a prime or zero.
12. Let $U = \{0, 3\}$ be an ideal of the ring Z_6 , then write the quotient ring Z_6/U

Unit-II

13. If f is a homomorphism of a ring R into a ring R' , then prove that f is an into isomorphism iff $\text{Ker } f = \{0\}$.
14. Prove that every quotient ring of a ring is a homomorphic image of the ring.
15. State and prove fundamental theorem of homomorphism.
16. If U_1, U_2 are two ideals of a ring R , then prove that $U_1 + U_2 = \{x + y : x \in U_1, y \in U_2\}$ is also an ideal of R .

17. If R/U is the quotient ring, then prove that
- R/U is commutative, if R is commutative
 - R/U has unity element, if R has unity element
18. Prove that the homomorphic image of a ring is a ring.
19. Prove that the homomorphic image of a commutative ring is a commutative ring.
20. If f is a homomorphism of a ring R into a ring R' , then prove that $\text{Ker } f$ is an ideal of R
21. If a function $f: C \rightarrow R$ is defined by $f(m + in) = m$, $m + in \in C$, is a homomorphism.

Unit-III

22. $a = x + y + z$, $b = x^2 + y^2 + z^2$, $c = xy + yz + zx$ then prove that $[\text{grad } a, \text{grad } b, \text{grad } c] = 0$.
23. Find the directional derivative of $\phi = x^2yz + 4xz^2$ in the direction of vector $2i - j - 2k$ at $(1, -2, -1)$.
24. Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at $P(1, 2, 3)$ in the direction of the line PQ , where $Q = (5, 0, 4)$.
25. Show that, 1. $\nabla^2 \left(\frac{x}{r^3} \right) = 0$, 2. $\nabla^2 (\log r) = \frac{1}{r^3}$.
26. If $\vec{r} = xi + yj + zk$ and a is a constant vector, then prove that $\frac{\partial}{\partial x}(a \cdot r)i + \frac{\partial}{\partial y}(a \cdot r)j + \frac{\partial}{\partial z}(a \cdot r)k = a$.
27. Prove that $\nabla \left(\frac{1}{r} \right) = \frac{-\vec{r}}{r^3}$.
28. Find the greatest value (maximum) of the directional derivative of the function $f = x^2yz^3$ at $(2, 1, -1)$.

Unit-IV

29. Evaluate $\int_1^2 F(t) \cdot dt$, where $F(t) = (t - t^2)i + 2t^3j - 3k$.
30. Find $\int_C F \cdot d\vec{r}$, where $F = xyi + yzj + zzk$ and the curve C is $\vec{r} = ti + t^2j + t^3k$, t varying from -1 to 1 .
31. Find $\int_C F \cdot d\vec{r}$, where $F = 3x^2i + (2xz - y)j + zk$ along the straight line C from $(0,0,0)$ to $(2,1,3)$.
32. If $F = 3xyi - y^2j$, evaluate $\int_C F \cdot d\vec{r}$, where C is the curve $y = 2x^2$ in xy -plane from $(0,0)$ to $(1,2)$.
33. Evaluate $\oint_C F \cdot N \, dS$, where $F = zi + xj - 3y^2zk$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.
34. If $F = 2xzi - xj + y^2k$, evaluate $\int_V F \cdot dV$, where V is the region bounded by the surfaces $x = 0, x = 2, y = 0, y = 6, z = x^2, z = 4$.

Unit-V

35. State and prove Green's identities
36. Show that $\int_S (axi + byj + czk) \cdot N \, dS = 4 \frac{\pi}{3} (a + b + c)$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

37. Compute $\int_S (ax^2 + by^2 + cz^2) dS$ over the sphere $x^2 + y^2 + z^2 = 1$.

38. Apply divergence theorem to evaluate $\iiint_S (x+y) dy dz + (y+z) dz dx + (x+y) dx dy$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$.

39. Evaluate $\int_S F \cdot N ds$, where $F = 2x^2yi - y^2j + 4xz^2k$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $z = 2$.

40. Evaluate $\int_C (\cos x \sin y - xy) dx + \sin x \cos y dy$ by Green's theorem where C is the circle $x^2 + y^2 = 1$.

41. If $F = yi + (x - 2xz)j - xyk$, evaluate $\int_S (\nabla \times F) \cdot N dS$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy - plane.

42. Prove that $\text{Curl grad } \phi = 0$ by using Stokes theorem

Essay Questions

Unit-I

1. Let Z be the set of all integers, then prove that $(Z, *, o)$ is a commutative ring with unity, where the operations $*$ and o are defined by $a * b = a + b - 1$ and $aob = a + b - ab$.
2. Prove that, every finite integral domain is a field.
3. If p is a prime, then prove that Z_p , the ring of residue classes modulo p is a field.
4. Let S be a non empty subset of a ring R . Then prove that S is a subring of R iff $a - b \in S$ and $ab \in S$, for all $a, b \in S$.
5. Prove that every field has no proper ideals.
6. Prove that, a commutative ring R with unity element is a field, if R has no proper ideals.
7. If U_1 and U_2 are two ideals of a ring R , then prove that $U_1 \cup U_2$ is an ideal of R iff $U_1 \subseteq U_2$ or $U_2 \subseteq U_1$.
8. Prove that the ring of integers Z is a principal ideal ring.
9. If U is an ideal of the ring R , then the set $R/U = \{x + U : x \in R\}$ is a ring w.r.t. the induced operations of addition (+) and multiplication (.) of cosets defined follows $(a + U) + (b + U) = (a + b) + U$ and $(a + U) \cdot (b + U) = (a \cdot b) + U$.

Unit-II

11. In the ring Z of integers, prove that an ideal generated by prime integer is a maximal ideal.
12. If M is a maximum ideal of the ring of integers Z , then prove that M is generated by a prime number.
13. Prove that, an ideal U of a commutative ring R with unity is maximal iff the quotient ring R/U is a field.
14. Prove that, an ideal $U \neq R$ of a commutative ring R , is a prime ideal iff the quotient ring R/U is an integral domain.
15. If $f: R \rightarrow R'$ be an isomorphism from the ring R to the ring R' , then prove that
 - i. $f(0) = 0'$, where 0 and $0'$ are the zero elements of R and R' .
 - ii. $f(-a) = -f(a)$, $\forall a \in R$.
 - iii. R' is a commutative, if R is commutative.
 - iv. R' is an integral domain, if R is an integral domain.
 - v. R' is a field, if R is a field.

16. Let $Z\sqrt{2} = \{m + n\sqrt{2} : m, n \in Z\}$ be a ring under addition and multiplication of numbers. Prove that $f: Z\sqrt{2} \rightarrow Z\sqrt{2}$ defined by $f(m + n\sqrt{2}) = m - \sqrt{2}$, $\forall m + n\sqrt{2} \in Z\sqrt{2}$ is an automorphism. Also find $\text{Ker } f$.
17. Prove that, every maximal ideal of a commutative ring R with unity is a prime.
18. Prove that a field has no proper homomorphic image.

Unit-III

19. Find $\text{div } f$ and $\text{curl } f$, where $f = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$ at $(1, -1, 1)$.
20. Prove that, 1. $\text{div } \vec{r} = 3$, 2. $\text{curl } \vec{r} = 0$, 3. $\text{div}(\vec{r} \times \vec{a}) = 0$, 4. $\text{curl}(\vec{r} \times \vec{a}) = -2\vec{a}$.
21. Prove that $\text{grad}(A \cdot B) = (B \cdot \nabla)A + (A \cdot \nabla)B + B \times \text{curl } A + A \times \text{curl } B$.
22. Prove that $\text{div}(A \cdot B) = B \cdot \text{curl } A - A \cdot \text{curl } B = (\nabla \times A) \cdot B - (\nabla \times B) \cdot A$.
23. Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at the point $(1, 1, 1)$.

Unit-IV

24. If $F = (x^2 + y^2)i - 2xyj$, evaluate $\oint_C F \cdot dr$, where the curve C is the rectangle in the xy -plane bounded by $y=0, y=b, x=0, x=a$.
25. If $F = 4xzi - y^2j + yzk$, evaluate $\int_S F \cdot N ds$ where S is the surface of the cube bounded by $x = 0, x = a, y = 0, y = b, z = 0, z = c$.
26. If $F = (x + y^2)i - 2xj + 2yxk$, evaluate $\int_S F \cdot N ds$, where S is the surface of the plane from $2x + y + 2z = 6$ in the first octant.
27. Find $\int_S F \cdot N ds$ over the entire surface of the region bounded by $x^2 + z^2 = 9, x = 0, y = 0, z = 0$ and $y = 8$, if $F = 6zi + (2x + y)j - xk$.
28. If $\phi = 45x^2y$, evaluate $\iiint_V \phi dV$, where V is the closed region bounded by the plane $4x + 2y + z = 8, x = 0, y = 0, z = 0$

Unit-V

29. State and prove Gauss's divergence theorem.
30. Verify Gauss's divergence theorem to evaluate $\int_S ((x^3 - yz)i - 2x^2yj + zk) \cdot N dS$ over the surface of the cube bounded by the coordinate planes $x = y = z = a$.
31. State and prove Green's theorem.
32. Verify Green's theorem in the plane for $\int_C (xy + y^2) dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.
33. State and prove Stokes theorem.
34. Verify Stokes theorem for $F = (y - z + 2)i + (yz + 4)j - xzk$ where S is the surface of the cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the xy -plane.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
III B.Sc., MATHEMATICS – Semester V (w.e.f. 2018-19)
Paper VI Course: Linear algebra

Total Hrs. of Teaching-Learning: 75 @ 5 h / Week

Total Credits: 05

Objective:

- To improve the students ability of understanding the most application oriented topic in Mathematics that is Linear Algebra.
- To equip the skill of understanding the concepts and writing the proofs of the Theorems.

Unit - I: Vector Spaces – I

(15 Hrs)

Vector spaces, General properties of vector spaces, n-dimensional vectors, Addition and scalar multiplication of vectors, Internal and external composition, Null Space, Vector Subspaces, Algebra of subspaces, Linear sum of two subspaces, Linear combination of vectors, Linear span, Linear dependence and linear independence of Vectors.

Unit - II: Vector spaces – II

(15 Hrs)

Basis of vector space, Finite dimensional vector space, Basis extension, Co-ordinates, Dimension of vector space, Dimension of subspace, Quotient space and Dimension of Quotient space.

Unit - III: Linear transformations

(15 Hrs)

Linear transformations, Linear operators, Properties of linear transformation, Sum and product of linear transformations, Algebra of Linear Operators, Range space and NullSpace of LT, Rank and Nullity of a LT, Rank & Nullity theorem.

Unit - IV: Matrix

(15 Hrs)

Linear Equations, Characteristic Values and Characteristic Vectors of square matrix – Cayley - Hamilton Theorem.

Unit - V: Inner Product Space

(15 Hrs)

Inner Product spaces, Euclidean and Unitary spaces, Norm or length of a vector, Schwartz's inequality, Triangle Inequality, Parallelogram law, Orthogonality and orthonormal set, Complete orthonormal set, Gram-Schmidt Orthogonalisation Process, Bessel's inequality and Parsvels identity.

Prescribed Text Books:

J.N. Sharma & A.R. Vasista, Linear Algebra, Krishna Prakasham Mandir, Meerut.

Books for Reference:

1. III year Mathematics Linear Algebra and Vector Calculus, Telugu Academy.
2. A Text Book of B.Sc. Mathematics, Vol-III, S. Chand & Co.

**BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-V, PAPER VI**

Unit	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Vector spaces - I	01	01	02	22
II	Vector spaces - II	01	01	02	22
III	Linear Transformation	01	01	01	14
IV	Char. values and char. vectors	01	01	02	22
V	Inner product spaces	01	01	01	14
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
 S.A.Q. = Short answer questions (5 marks)
 E.Q. = Essay questions (8 marks)

Very Short answer questions : $5 \times 1 M = 05$
 Short answer questions : $3 \times 5 M = 15$
 Essay questions : $5 \times 8 M = 40$

Total Marks : _____ = 60

P.R. Govt. College (Autonomous), Kakinada
III year B.Sc. Degree Examinations, – V Semester
Mathematics Course: Linear Algebra
Paper-VI (Model Paper w.e.f. 2019-20)

Time: 2 Hrs 30 Min

Max. Marks: 60 M

PART – I

Answer ALL the following questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Define linear combination of vectors.
2. Write the standard basis of $V_2(R)$
3. Find the null space of the transformation $T : R^2 \rightarrow R^3$ defined by $T(x, y) = (x + y, x - y, y)$

4. Find the Eigen values of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

5. Write Bessel's inequality.

PART – II

Answer any THREE of the following questions. Each question carries 5 marks. 3 x 5 = 15 M

6. Determine whether the set of vector $\{(1, -2, 1), (2, 1, -1), (7, -4, 1)\}$ is linearly dependent or Linearly independent.
7. If W is a subspace of a finite dimensional vector space $V(F)$ then prove that W is also finite dimensional and $\dim W \leq \dim V$.
8. Find $T(x, y, z)$ where $T: R^3 \rightarrow R$ is defined by $T(1,1,1)=3, T(0,1,-2)=1, T(0,0,1)=-2$.
9. Solve the system of linear equations $2x - 3y + z = 0, x + 2y - 3z = 0, 4x - y - 2z = 0$.
10. State and prove Parseval's identity.

PART – III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks.

5 X 8 = 40 M

SECTION – A

11. Prove that a non empty subset W of a vector space $V(F)$ is a subspace of V if and only if $a, b \in F, \alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$.
12. Let $V(F)$ be a vector space and $S = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ is a finite subset of non-zero vectors of $V(F)$. Then S is linearly dependent if and only if some vector $\alpha_k \in S, 2 \leq k \leq n$, can be expressed as a linear combination of its preceding vectors.
13. Show that the set of vectors $\{(2, 1, 4), (1, -1, 2), (3, 1, -2)\}$ form a basis for R^3 .
14. Let W be a sub space of a finite dimensional vector space $V(F)$, then prove that $\dim V/W = \dim V - \dim W$.

SECTION – B

15. State and prove rank and nullity theorem.
 16. Discuss for all values of λ , the system of equations $x + y + 4z = 6, x + 2y - 2z = 6,$
 $\lambda x + y + z = 6$ as regards existence and nature of solutions.
 17. State and prove Cauchy-Schwarz's inequality.
 18. Applying Gram-Schmidt process, obtain an orthonormal basis of $\mathbb{R}^3(\mathbb{R})$ from the basis
11. $\{(2, 0, 1), (3, -1, 5), (0, 4, 2)\}.$
 - 12.

PR GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank
PAPER-VI: LINEAR ALGEBRA

Very short answers

UNIT-I

1. Define addition of vectors and scalar multiplication of vectors.
2. Define internal composition and external composition.
3. Define vector space and null space (zero vector space).
4. Define linear combination of vectors.
5. Define linear span of a set.
6. A vector space $V(F)$ is said to be finite dimensional vector space if it has _____.
7. If S and T are the subspaces of vector space $V(F)$ then $L(S \cup T) =$ _____.
8. Show that the vectors $(2, -3), (6, -9)$ are linearly dependent in $V_2(\mathbb{R})$.

UNIT-II

9. Define basis of a vector space.
10. Define finite dimensional vector space.
11. Write the standard basis of $V_2(\mathbb{R})$
12. Show that the vectors $(1, -1), (-2, -3)$ form a basis of $V_2(\mathbb{R})$.
13. Extend $S = \{(1, 1, 1)\}$ into a basis for $V_3(\mathbb{R})$.
14. What is the dimension of $V_3(\mathbb{R})$?

UNIT-III

15. Define linear transformation.
16. Define null space and range space.
17. Find the null space of the linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by
 $T(x, y) = (x + y, x - y, y)$
18. Test whether the mapping $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by
 $(T(a, b)) = (2a + 3b, 3a - 4b)$ is a linear transformation or not.
19. Test whether the mapping $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by
 $T(a, b) = (|a|, 0)$ is a linear transformation or not.

UNIT-IV

20. Define Eigen values of a matrix.
21. Define Eigen vectors of a square matrix.
22. Define diagonal matrix.
23. Define skew symmetric matrix.

24. Find the Eigen values of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

UNIT-V

25. Define an inner product space.
26. Define unitary space.
27. Define Euclidian space.
28. State parallelogram law.
29. State triangle inequality.
30. State Cauchy-Schwartz's inequality.
31. State Bessel's inequality.
32. State Parseval's identity.
33. Every orthogonal set of non zero vectors in an inner product space $V(F)$ is _____ set.
34. Find a unit vector which is orthogonal to $(1,2,1)$ in $R^3(R)$ with standard inner product.

Short answers

UNIT-I

1. Let V be the set of all pairs (a, b) of real numbers. Show that V is not a vector space with the operation $(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, 0)$, $c(a_1, b_1) = (ca_1, b_1)$.
2. Let p, q, r be the fixed elements of a field F . Show that the set W of all triads (x, y, z) of elements of F such that $px + qy + rz = 0$ is a vector space of $V_3(F)$.
3. Let R be the field of real numbers and $W = \{(x, y, z) : x, y, z \text{ are rational numbers}\}$. Is W is a subspace of $V_3(R)$?
4. Show that the subset $W = \{(a, b, c) / a^2 + b^2 + c^2 \leq 1\}$ is not a subset of $R^3(R)$.
5. Prove that the linear span $L(S)$ of any subset S of a vector space $V(F)$ is a subspace of $V(F)$.
6. If α, β, γ are linearly independent vectors of $V(R)$, then show that $\alpha + \beta, \beta + \gamma, \gamma + \alpha$ are also linearly independent.

UNIT-II

7. Show that the set of vectors $\{(2, 1, 4), (1, -1, 2), (3, 1, -2)\}$ form a basis for R^3 .
8. Show that the set $S = \{(1, 0, 0), (0, 1, 0), (1, 1, 0), (1, 1, 1)\}$ is a spanning set of $R^3(R)$ but not a basis.
9. Prove that every set of $(n + 1)$ or more vectors in an n dimensional vector space is linearly dependent.
10. Let W_1 and W_2 be two subspaces of R^4 given by $W_1 = \{(a, b, c, d) : b - 2c + d = 0\}$, $W_2 = \{(a, b, c, d) : a = d, b = 2c\}$. Find the basis and dimension of (i) W_1 (ii) W_2 (iii) $W_1 \cap W_2$ and hence find $\dim(W_1 + W_2)$.
11. Let W be a subspace of a finite dimensional vector space $V(F)$, then prove that $\dim V/W = \dim V - \dim W$.

UNIT-III

12. Define linear transformation and show that the function $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x - y, 0, y + z)$ is a linear transformation.
13. Find a linear transformation $T: R^3 \rightarrow R$ such that $T(1, 1, 1) = 3, T(0, 1, -2) = 1, T(0, 0, 1) = -2$.
14. Let $T: R^3 \rightarrow R^3$ be defined by $T(x, y, z) = (x - y + 2z, 2x + y - z, -x - 2y)$, then verify Rank-nullity theorem.
15. Describe explicitly the linear transformation $T: R^3 \rightarrow R^3$ whose range space is spanned by $\{(1, 0, -1), (1, 2, 2)\}$.

UNIT-IV

16. Solve the system of linear equations $2x - 3y + z = 0, x + 2y - 3z = 0, 4x - y - 2z = 0$ are consistent or not.
17. Show that the system of linear equations $x - 4y + 7z = 14, 3x + 8y - 2z = 13, 7x - 8y + 26z = 5$ are inconsistent.
18. Find the eigen values and eigen vectors of the square matrix $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 4 \end{pmatrix}$.
19. State and prove Cayley-Hamilton theorem.
20. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} -2 & 1 & -1 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{pmatrix}$.

UNIT-V

21. State and Prove Triangle inequality.
22. State and prove Parallelogram law in an inner product space $V(F)$.
23. State and prove Parseval's identity in an inner product space $V(F)$.
24. Prove that every orthogonal set of non zero vectors in an IPS $V(F)$ is linearly independent.
25. Prove that every orthonormal set of vectors is linearly independent.
26. Prove that the set $S = \left\{ \left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3} \right), \left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3} \right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right) \right\}$ is an orthonormal set in the inner product space $R^3(R)$ with the standard inner product.

Essay questions

UNIT-I

1. Let $V(F)$ be a vector space and $W \subseteq V$. Then prove that the necessary and sufficient conditions for W to be a subspace of V are
- (i) $\alpha \in W, \beta \in W \Rightarrow \alpha - \beta \in W$ (ii) $\alpha \in W, \alpha \in W \Rightarrow \alpha\alpha \in W$

2. State and prove the necessary and sufficient condition for a non-empty subset of a vector space to be a subspace.
3. Prove that the union of two subspaces of a vector space is a subspace if and only if one is contained in the other.
4. If W_1 and W_2 are two subspaces of a vector space $V(F)$ then prove that
 - (i) $W_1 + W_2$ is a subspace of $V(F)$ and (ii) $W_1 \subseteq W_1 + W_2$ and $W_2 \subseteq W_1 + W_2$.
5. If S and T are the subsets of a vector space $V(F)$ then prove that
 - (i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$ and (ii) $L(S \cup T) = L(S) + L(T)$.
6. Let $V(F)$ be a vector space and $S = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ is a finite subset of nonzero vectors of $V(F)$. Then prove that S is linearly dependent if and only if some vector $\alpha_k \in S, 2 \leq k \leq n$, can be expressed as a linear combination of its preceding vectors.

UNIT-II

7. Let W_1 and W_2 be two subspaces of a finite dimensional vector space $V(F)$. Then prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.
8. State and Prove Basis Existence theorem.
9. State and prove Basis extension theorem.
10. Prove that any two bases of a finite dimensional vector space $V(F)$ have the same number of elements.
11. Define dimension of a finite dimensional vector space. If W is a subspace of a finite dimensional vector space $V(F)$ then prove that W is finite dimensional and $\dim W \leq n$.
12. Let W be a subspace of a finite dimensional vector space $V(F)$ then prove that $\dim V/W = \dim V - \dim W$.

UNIT-III

13. Let $L(U, V)$ be the vector space of all linear transformations from $U(F)$ to $V(F)$ such that $\dim U = n$ and $\dim V = m$, then prove that $\dim L(U, V) = mn$.
14. Let $U(F)$ and $V(F)$ be two vector spaces. Let $T: U(F) \rightarrow V(F)$ be a linear transformation. Then define range set of T and prove that the range set $R(T)$ is a subspace of $V(F)$. If U is finite dimensional then $R(T)$ is finite dimensional.
15. Let $U(F)$ and $V(F)$ be two vector spaces. Let $T: U \rightarrow V$ be a linear transformation. Then define null space and prove that the null space $N(T)$ is a subspace of $U(F)$.
16. State and Prove Rank - nullity theorem of Linear Transformation.
17. Find the null space, range, rank and nullity of the transformation $T: R^2 \rightarrow R^3$ defined by $T(x, y) = (x + y, x - y, y)$.

UNIT-IV

18. Find the characteristic roots and the corresponding vectors of the matrix

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

19. Find the characteristic equation of A and hence find A^{-1} , where

$$A = \begin{pmatrix} 1 & 2 & -4 \\ 3 & -1 & 2 \\ 2 & 5 & 0 \end{pmatrix}.$$

20. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} -2 & 1 & -1 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{pmatrix}$.

UNIT-V

21. State and prove Cauchy-Schwarz's inequality.

22. If u and v are two vectors in a complex inner product space $V(F)$, then prove that

$$4 \langle u, v \rangle = \|u + v\|^2 - \|u - v\|^2 + i\|u + iv\|^2 - \|u - iv\|^2.$$

23. Given $\{(1, -1, 2), (0, 2, 1), (1, 2, 0)\}$ is a basis of $R^3(R)$. Construct an orthonormal basis using Gram-Schmidt orthogonalisation process.

25. In a real inner product space, if u and v are two vectors such that $\|u\| = \|v\|$, then prove that $u - v$ and $u + v$ are orthogonal.

P. R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

III B.Sc. MATHEMATICS – Semester VI (w.e.f. 2018-19)

Course (Elective VII (A)) : LAPLACE TRANSFORMS

Total Hours of Teaching-Learning : 75 @ 5 h/week

Total credits : 05

Objectives:

- To understand the concepts of Laplace Transform and Inverse Laplace Transform.
- To find the Laplace transform of some functions.

UNIT – I : Laplace Transform I (15 hrs)

Definition of Integral Transform – Laplace Transform Linearity, Piecewise continuous Functions, Existence of Laplace Transform, Functions of Exponential order, and of Class A.

UNIT – II : Laplace Transform II (15 hrs)

First Shifting Theorem, Second Shifting Theorem, Change of Scale Property, Laplace Transform of the derivative of $f(t)$, Initial Value theorem and Final Value Theorem.

UNIT – III : Laplace Transform III (15 hrs)

Laplace Transform of Integrals – Multiplication by t , Multiplication by t^n – Division by t , Laplace Transform of Bessel's function, Laplace Transform of error function, Laplace Transform of Sine and cosine integrals.

UNIT – IV : Inverse Laplace Transform I (15 hrs)

Definition of Inverse Laplace Transform. Linearity, First Shifting Theorem, Second Shifting Theorem, Change of Scale property, use of partial fractions, Examples.

UNIT – V : Inverse Laplace Transform II (15 hrs)

Inverse Laplace transforms of Derivatives – Inverse Laplace Transforms of Integrals – Multiplication by powers of p – Division powers of ' p '- Convolution definition - Convolution Theorem – proof and Applications – Heaviside's Expansion theorem and its Applications.

Prescribed Text book:

Integral Transforms by A.R.Vasishta and R.K. Gupta, Krishnaprakashan media Pvt. Ltd. Meerat.

Reference Books:

1. Integral Transforms by Dr. J. K. Goyal and K. P. Gupta, Pragati Prakashan.
2. M. D. Raisinghanian Integral Transform, S. Chand & Co., New Delhi.

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-VI
PAPER –VII, Elective VII (A)

UNIT	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Laplace Transforms - I	01	01	01	14
II	Laplace Transforms - II	01	01	02	22
III	Laplace Transforms - III	01	01	01	14
IV	Inverse Laplace Transforms - 1	01	01	02	22
V	Inverse Laplace Transforms - 2	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q. = Essay questions (8 marks)

Very Short answer questions : $5 \times 1 M = 05$
Short answer questions : $3 \times 5 M = 15$
Essay questions : $5 \times 8 M = 40$

Total Marks : = 60

P. R. Government College (Autonomous), Kakinada
 III Year B.Sc. Degree Examinations, VI Semester - Mathematics
 Course (Elective VII (A)) - LAPLACE TRANSFORMS
 PAPER - VII : MODEL PAPER (W.e.f. 2019-20)

Time: 2 Hrs 30 Min

Max. Marks : 60 M

PART - I

Answer ALL the following questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Define Laplace Transform.
2. Find $L[t^3 e^{-3t}]$.
3. What is the Laplace transform of $L\left\{\frac{\sin t}{t}\right\}$
4. Write the Inverse of Laplace Transform of $\frac{a}{p^2+a^2}$.
5. If $L^{-1}\{f(p)\} = F(t)$ then what is the inverse Laplace transform of $f^{(n)}(p)$?

PART - II

Answer any THREE of the following questions. Each question carries 5 marks. 3 x 5 = 15 M

6. Find $L\{t^n\}$, n is a positive integer.
7. State and Prove first shifting theorem in Laplace Transforms.
8. Find $L\{t(3\sin 2t - 2\cos 2t)\}$
9. Find $L^{-1}\left\{\frac{3p-2}{p^2-4p+20}\right\}$.
10. Find $L^{-1}\left\{\log\left(1 + \frac{1}{p^2}\right)\right\}$.

PART - III

Answer any FIVE questions from the following by choosing at least TWO from each section.

Each question carries 8 marks.

5 X 8 = 40 M

SECTION - A

11. Find $L\{F(t)\}$, where $F(t) = \begin{cases} 0 & \text{when } 0 < t < 1 \\ t & \text{when } 1 < t < 2 \\ 0 & \text{when } t > 2 \end{cases}$
12. Find $L\{\sin\sqrt{t}\}$.
13. If $L\{F(t)\} = f(p)$ then prove that $L\{F(at)\} = \frac{1}{a} f\left(\frac{p}{a}\right)$.
14. Find $L\{C_i(t)\}$

SECTION - B

15. Prove that $L^{-1}\left\{\frac{4p+5}{(p-1)^2(p+2)}\right\} = 3te^t + \frac{1}{3}e^t - \frac{1}{3}e^{-2t}$
16. Show that $L^{-1}\left\{\frac{p^2}{(p^4+4a^4)}\right\} = \frac{1}{2a}(\cosh at \cdot \sin at + \sinh at \cdot \cos at)$.
17. Apply convolution theorem to find the inverse Laplace transform of the function $\frac{1}{(p-2)(p^2+1)}$.
18. Find $L^{-1}\left\{\frac{p}{(p^2+a^2)^2}\right\}$.

P. R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

III B.SC MATHEMATICS – Semester VI (w.e.f. 2018-19)

Course (Elective - VII (B)): Numerical Analysis

Total Hrs. of Teaching-Learning: 75 @ 5 h / Week

Total Credits: 05

Objective:

- To find the different types of errors in computation and then to reduce the errors.
- To find the approximate Polynomial for the given data when the data is even or uneven by using interpolation, also we can find the differentiation even if the function is not known explicitly.
- To find the solution of Algebraic and Transcendental equations using Bisection, Falsi Position, Iteration and Newton Raphson methods.

Unit - I: Errors in Numerical Computation (15 hrs)

Errors and their accuracy, Mathematical preliminaries, Errors and their analysis, Absolute, Relative and Percentage errors, A general error formula, Errors in a series approximation.

Unit - II: Solutions of Algebraic and transcendental equations (15 hrs)

Bisection Method, Iteration Method, Method of false position, Newton Raphson Method, Generalised Newton Raphson method, Muller's method.

Unit - III: Interpolation – I (15 hrs)

Errors in polynomial interpolation, Finite Differences, Forward, Backward and central difference operators, Shift and average difference operators, Symbolic relation between the operators, Detection of errors by use of difference tables, Differences of a polynomial.

Unit - IV: Interpolation - II (15 hrs)

Interpolation for equal intervals: Newton's forward, backward, Gauss forward, Backward, Strirling's, Bessel's and Everette's formulae.

Unit - V: Interpolation – III (15 hrs)

Interpolation for uneven intervals: Lagrange's interpolation formula, Error in Lagrange's Interpolation, Divided differences and their properties, Relation between divided differences formula, Forward, Backward and central difference operators, Newton's divided differences, Inverse Interpolation.

Prescribed Text books:

Numerical Analysis by S. Ranganatham, MVSSN Prasad, Dr. V. Ramesh Babu, S. Chand & Company.

Reference books:

1. Numerical Analysis by S. S. Sastry, Prentice Hall, NewDelhi.
2. Numerical Analysis by Kamali Surya Narayana, S. Chand & co, New Delhi.
3. Numerical Analysis by Gupta & Malik, Krishna Prakashan media (P) Ltd, Meerut.S

P.R. Govt. College (Autonomous), Kakinada
III year B.Sc. Degree Examinations : VI Semester - Mathematics
Course (Elective VII (B)) : Numerical Analysis
Paper VII: MODEL PAPER (w.c.f. 2019-20)

Time: 2 Hrs 30 Min

Max. Marks : 60 M

PART - I

Answer ALL the following questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Estimate $1/3$ to three significant digits and find its absolute error.
2. Write the convergent condition for iterative method.
3. Define Shift operator.
4. Write the Bessel's Formula for interpolation
5. Write the divided difference of $f(x) = x^2 - 5$ for the arguments 2 and 4.

PART -II

Answer any THREE of the following questions. Each question carries 5 marks. **3 x 5 = 15 M**

6. Define absolute, relative and percentage errors and give an example.
7. Find a root of the equation $x^3 - 2x - 5 = 0$ by using Newton-Raphson method.
8. Find the missing term in the following data given below.

X	0	1	2	3	4
Y	1	3	9	-	81

9. Derive Newton's forward interpolation formula.
10. Using the inverse Lagrange's Interpolation Formula, if $y_1 = 4$, $y_3 = 12$, $y_4 = 19$, $y_x = 7$, then find the value of x .

PART -III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks.

5 X 8 = 40 M

SECTION - A

11. If $u = 4x^2y^3/z^4$ and errors in x, y, z be 0.001, compute the relative maximum error in u , when $x = y = z = 1$.
12. Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to four significant digits and find its absolute and relative errors.
13. Find the real root of the equation $x^3 - 9x + 1 = 0$ by using Regula Falsi Method.
14. Find the root of the equation $f(x) = e^x - 3x$ by using Newton-Raphson method.

SECTION - B

15. Prove that $(\frac{\Delta^2}{E})e^x \cdot (\frac{Ee^x}{\Delta^2 e^x}) = e^x$, the interval of differencing being unit.
16. Using Newton's Forward interpolation formula, find the value of $f(x)$ when $x = 1.4$.

X	1.1	1.3	1.5	1.7	1.9
Y	0.21	0.69	1.25	1.89	2.61

17. Apply Stirling formula to find the value of $f(1.22)$ from

x	0	0.5	1.0	1.5	2.0
f(x)	0	0.191	0.341	0.433	0.477

18. By means of Newton's divided difference formula, find the value $f(8)$ and $f(15)$ from the following table.

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

PR GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
Question Bank for
PAPER-VII: NUMERICAL ANALYSIS

Very Short Answer Questions

Unit-I

1. Define absolute, relative, percentage errors.
2. Find the relative error, if $\frac{1}{3}$ is approximated to 0.333.
3. Find the percentage error, if 625.483 is approximated to 3 significant figures.
4. Round off the numbers 37.46235 to four significant digits and compute E_A, E_R, E_P .
5. Round off the numbers 865250 to four significant digits and compute E_A, E_R, E_P .
6. Write general error formula.

Unit-II

7. Find the interval in which $\sqrt{29}$ lies _____.
8. Write the formula for Newton - Raphson method to find the approximate root.
9. Find the interval in which a root of the equation $x^3 - 4x - 9 = 0$ lies.
10. Write the condition for the convergence of iterative method.
11. Transcendental equation consists of _____ functions.
12. If $f(a) < 0, f(b) > 0$ then by bisection method the first approximation for the root of $f(x) = 0$ is _____.
13. Rate of convergence of Newton-Raphson method is _____.
14. Oldest method for finding root of $f(x) = 0$ is _____.

Unit-III

15. Prove that $\Delta = E - 1$.
16. Define shift operator.
17. Define forward difference operator, backward difference operator.
18. Define averaging operator.
19. Define enlargement operator.
20. Define inverse operator
21. Prove that $\nabla = \Delta E^{-1}$.
22. Evaluate $\Delta \frac{2^x}{x!}$ if $h=1$.
23. Construct the difference table for the following data.

x	3	3.1	3.2	3.3
y	0.3333	0.32258	0.31250	0.30303

24. Prove that $E - 1 = \Delta$
25. Evaluate $\Delta \tan^{-1} ax$.
26. Evaluate Δe^x if $h=1$.
27. Evaluate $\Delta \tan^{-1} x$
28. Evaluate $\Delta \log f(x)$.
29. Evaluate $(\frac{\Delta^2}{E})x^3$.
30. Construct the difference table for the following data:

x	1	2	3	4	5
y	12	16	15	18	20

31. Show that the first forward difference of a constant is zero.

Unit-IV

32. Write the Newton's forward interpolation formula.
33. Write the Newton's backward interpolation formula.
34. Write the Gauss forward interpolation formula.
35. Write the Gauss backward interpolation formula.
36. Write the Bessel's formula for interpolation.
37. Write the Stirling's interpolation formula
38. Write the Everett's interpolation formula.

Unit-V

39. Write the Lagrange's interpolation formula.
40. Write the relation between divided difference and forward differences.
41. Write the relation between divided difference and backward differences.
42. Define inverse interpolation.
43. Write the divided difference of $f(x) = x^2 - 5$ for the arguments 2, 4.

Short Answer Questions

Unit-I

1. Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to four significant digits and find its absolute and relative errors.
2. Define absolute, relative, percentage errors and give an example.
3. Derive general error formula.

4. When the numbers $x = 4.488$, $y = 1.321$ are rounded to two decimal places, find the value of $q = \frac{x}{y}$ and $E_R(q)$.

Unit-II

5. Explain bisection method.
6. Find the root of the equation $x^3 - 4x - 9 = 0$ using bisection method in 4 stages.
7. Explain iterative method.
8. Solve the equation $\sin x = 5x - 2$ by using iteration method.
9. Explain regular falsi method.
10. By using Regula - Falsi method, find an approximate root of the equation $x^4 - x - 10 = 0$ that lies in between 1.8 and 2. Carry out three approximations.
11. Explain Newton-Raphson method.
12. Find by Newton's method, the real root of the equation $xe^x - 2 = 0$ correct to 3 decimal places.
13. Explain Muller's method.

Unit-III

14. Prove that (i) $\Delta f(x)g(x)$, (ii) $\Delta \frac{f(x)}{g(x)}$ and (iii) $E = e^{hD}$.
15. Find the missing term in the following data.

x	0	1	2	3	4
y	1	3	9	—	81

16. Find the missing term in the following data.

x	1	2	3	4	5	6	7
y	2	4	8	---	32	64	128

17. The following table gives a set of values of x and the corresponding values of $y = f(x)$.

x	10	15	20	25	30	35
y	19.97	21.51	22.47	23.52	24.65	25.89

Form the forward difference table and write down the values of $\Delta f(10)$, $\Delta^2 f(10)$, $\Delta^3 f(15)$ and $\Delta^4 f(15)$.

Unit-IV

18. Derive Newton's forward interpolation formula.
19. Find the polynomial which satisfies the data $y(0) = 1$, $y(1) = 0$, $y(2) = 1$, $y(3) = 10$ and hence find $y(4)$.
20. Find $\sin 52'$ from the data.

x	$45'$	$50'$	$55'$	$60'$
$\sin x$	0.7071	0.7660	0.8192	0.8660

21. Derive Newton's backward interpolation formula.
22. Find $f(7)$ from the following table.

x	0	2	4	6	8
$f(x)$	7	13	43	45	367

23. Derive Gauss forward interpolation formula.
24. Find $f(32)$ from the following table by Using Gauss forward interpolation formula.

x	25	30	35	40
$f(x)$	0.2707	0.3027	0.3386	0.3940

25. Derive Gauss backward interpolation formula.
26. Estimate the sales of the year 1936 from the following table

Year	1901	1911	1921	1931	1941	1951
Sales in lakhs	12	15	20	27	39	52

27. Apply Strilling's formula to find the value of $f(2.2)$ from the following data.

x	0	0.5	1.0	1.5	2.0
$f(x)$	0	0.191	0.341	0.433	0.477

28. Apply Bessel's formula to find a polynomial of degree three or less which takes the following values of the function u_x

x	4	6	8	10
u_x	1	3	8	20

Unit-V

29. Derive Lagrange's interpolation formula.
 30. Derive Newton's divided difference interpolation formula.
 31. Find y at $x = 10$ from the data using Lagrange's interpolation formula

x	5	6	9	11
$f(x)$	12	13	14	16

32. Find u_x in the power of $(x - 1)$ from the data $u_0 = 8, u_1 = 11, u_4 = 68, u_5 = 123$.

Essay Questions

Unit-I

1. If $u = \frac{4x^2y^3}{z^4}$ and errors in x, y, z be 0.001, compute the relative maximum error in u , when $x = y = z = 1$.
2. If $u = \frac{5xy^2}{z^3}$ and errors in x, y, z be 0.001, compute the relative maximum error in u when $x = y = z = 1$.

Unit-II

3. Find the real root of $x = e^{-x}$ by iteration method.
4. Find the approximate value of real root of $x^3 - x - 1 = 0$ by bisection method.
5. Find out the roots of $x^3 - x - 4 = 0$ using false position method.
6. Evaluate value of $\sqrt{12}$ by iterative method.
7. Find the square root of 24 using Newton Raphson method.
8. Find the reciprocal of 18 using Newton Raphson method.
9. Find the square root of 25, given $x_0 = 2$ & $x_1 = 7$ using bisection method.
10. Find the root of the equation $x \log_{10}(x) = 1.2$ using false position method.

Unit-III

11. Prove that (i) $1 + \Delta = E$, (ii) $\nabla = 1 - E^{-1}$, (iii) $E = e^{hD}$, (iv) $\nabla = \Delta E^{-1}$,
 (v) $(1 + \Delta)(1 - \nabla) = 1$ and (vi) $\Delta \nabla = \nabla \Delta = \Delta - \nabla$.

12. Prove that (i) $\mu^2 = 1 + \frac{\delta^2}{4}$, (ii) $\Delta = \frac{\delta^2}{2} + \delta \sqrt{1 + \frac{\delta^2}{4}}$

(iii) $\nabla = 1 - e^{-hD}$ and (iv) $\Delta - \nabla = \delta^2$.

13. Find the missing term in the following table.

x	1	2	3	4	5	6	7	8
y	1	8	--	64	--	216	343	512

Unit-IV

14. Find $\log 58.75$ from the following data.

x	40	45	50	55	60	65
$\log x$	1.60206	1.65321	1.69897	1.74036	1.77815	1.81291

15. Estimate the population increase in between 1955-1961.

<i>year</i>	1921	1931	1941	1951	1961	1971
<i>Population in lakhs</i>	20	24	29	36	46	51

16. The following are marks obtained by 492 students in certain examination:

<i>marks not more than</i>	40	45	50	55	60	65
<i>no. of students</i>	210	253	307	381	413	492

Find the number of students who got more than 48 marks but not more than 50 marks.

17. Estimate the population of the year 1925.

<i>year</i>	1891	1901	1911	1921	1931
<i>Population (thousands)</i>	46	66	81	93	101

18. Using Newton's forward interpolation formula, find the value of $f(x)$ when $x = 1.4$

x	1.1	1.3	1.5	1.7	1.9
y	0.21	0.69	1.25	1.89	2.61

19. Interpolate by means of Gauss backward interpolation formula the sales for the concern for the year 1976, given that

<i>year</i>	1940	1950	1960	1970	1980	1990
<i>sales(in lakhs)</i>	17	20	27	32	36	38

20. Find $f(3.75)$ using Gauss forward interpolation formula, given that

x	2.5	3.0	3.5	4.0	4.5	5.0
$f(x)$	24.145	22.043	20.225	18.644	17.262	16.047

21. Given that $\sqrt{6500} = 80.6223$, $\sqrt{6510} = 80.6846$, $\sqrt{6520} = 80.7456$,
 $\sqrt{6530} = 80.8084$, find $\sqrt{6526}$ by using Gauss backward interpolation formula.
22. Apply Strilling's formula to find y_{28} given that $y_{20} = 49225$, $y_{25} = 48316$,
 $y_{30} = 47236$, $y_{35} = 45926$, $y_{40} = 44300$.
23. Apply Laplace-Everette's formula to obtain y_{25} given that $y_{20} = 2854$,
 $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$.

Unit-V

24. Using Lagrange's interpolation formula, calculate $f(3)$ for the following table.

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

25. By means of Newton's divided difference formula, find the values of $f(8)$, $f(15)$ from the following table.

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

26. Evaluate $f(10)$, given $f(x) = 168, 192, 336$ at $x = 1, 7, 15$, respectively by using Lagrange's interpolation formula.
27. Using divided difference table, find $f(x)$ which takes the values 1, 4, 40, 85 as $x = 0, 1, 3, 4$.

P.R.GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

III B.Sc. MATHEMATICS - Semester VI (w.e.f. 2018-19)

Course (Elective VII (C)) : NUMBER THEORY

Total Hours of Teaching - Learning : 75 @ 5 h/week

Total credits: 05

Objectives:

- To understand the concepts of Number Theory.
- To know about the applications of Number Theory

UNIT-I : Divisibility (15 hrs)

Divisibility – Greatest Common Divisor – Euclidean Algorithm – The Fundamental Theorem of Arithmetic.

UNIT-II : Congruences (15 hrs)

Congruences – Special Divisibility Tests - Chinese Remainder Theorem- Fermat's Little Theorem – Wilson's Theorem – Residue Classes and Reduced Residue Classes – Solutions of Congruences

UNIT-III : Number Theory from an Algebraic Viewpoint (15 hrs)

Number Theory from an Algebraic Viewpoint – Multiplicative Groups, Rings and Fields

UNIT-IV : Quadratic Residues (15 hrs)

Quadratic Residues - Quadratic Reciprocity – The Jacobi Symbol

UNIT-V Greatest Integer Function (15 hrs)

Greatest Integer Function – Arithmetic Functions – The Moebius Inversion Formula

Reference Books:

1. 'Introduction to the Theory of Numbers' by Niven, Zuckerman & Montgomery (John Wiley & Sons).
2. 'Elementary Number Theory' by David M. Burton.
3. 'Elementary Number Theory' by David, M. Burton, 2nd Edition, UBS Publishers.
4. 'Introduction to Theory of Numbers' by Davenport H., Higher Arithmetic, 5th Edition (John Wiley & Sons Publishers), Niven, Zuckerman & Montgomery.(Camb, Univ, Press)
5. 'Number Theory' by Hardy & Wright published by Oxford Univ, Press.
6. 'Elements of the Theory of Numbers' by Dence, J. B & Dence T.P published by Academic Press.

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SEMESTER-VI

PAPER –VII, Elective VII (C)

UNIT	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Divisibility	01	01	01	14
II	Congruence	01	01	02	22
III	Number Theory from an Algebraic view point	01	01	01	14
IV	Quadratic Residues	01	01	02	22
V	Greatest Integer Function	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)

S.A.Q. = Short answer questions (5 marks)

E.Q. = Essay questions (8 marks)

Very Short answer questions : $5 \times 1 M = 05$

Short answer questions : $3 \times 5 M = 15$

Essay questions : $5 \times 8 M = 40$

Total Marks : = 60

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
III Year B.Sc. Degree Examinations, VI Semester - MATHEMATICS
Course (Elective VII (C)) - NUMBER THEORY
PAPER VII, MODEL PAPER (w.c.f. 2019-20)

Time: 2 Hours 30 Min

Max. marks: 60 M

PART - I

Answer ALL the following questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Find the GCD of 18, 24.
2. Define 'a is congruent to b mod m'.
3. Define multiplicative group.
4. Define quadratic residues.
5. Define arithmetic function.

PART - II

Answer any THREE of the following questions. Each question carries 5 marks. 3 x 5 = 15 M

6. If $(a, b) = 1$ then prove that $(a+b, a-b)$ is either 1 or 2.
7. Solve the congruence $25x \equiv 15 \pmod{120}$
8. If $a \equiv b \pmod{m}, c \equiv d \pmod{m}$, then prove that $ac \equiv bd \pmod{m}$
9. If G is finite and $a \in G$ then prove that there is a positive integer $n < |G|$ such that $a^n = e$.
10. Prove that any complete residue system modulo m forms a group under addition modulo m .

PART - III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks.

5 X 8 = 40 M

SECTION - A

19. State and prove Fundamental theorem of arithmetic.
20. State and prove Fermat's little theorem.
21. If c/ab and $(b, c) = 1$ then prove that c/a .
22. Let $G=(a)$ be finite group of order n and let G' be a sub group of order m . Prove that m/n .

SECTION - B

19. Determine whether 219 is a quadratic residue or nonresidue mod 383.
20. Let p be an odd prime the prove that for all $n, \left(\frac{n}{p}\right) \equiv n^{p-1/2} \pmod{p}$
21. State and Prove mobius inversion formula.
22. Find all primes p such that $\left(\frac{10}{p}\right) = 1$.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
III B.Sc. MATHEMATICS - Semester VI (w.e.f. 2018-19)
Course (Elective VII (D)): GRAPH THEORY

Total Hours of Teaching-Learning : 75 @ 5 h/week

Total credits: 05

Objectives:

- To introduce the most application oriented field in Mathematics i.e., Graph Theory.
 - To impart the awareness on applications of Graph Theory
-

UNIT –I : Graphs and Subgraphs (15 hrs)

Graphs, Simple Graph, Graph isomorphism, The incidence and adjacency matrices, Subgraphs, Vertex degree, Hand shaking theorem, Paths and connection, Cycles.

UNIT -II : Applications, Trees (15 hrs)

Applications, The shortest path problem, Sperner's lemma, Trees, Cut edges and bonds, Cut vertices, Cayley's formula.

UNIT -III : Applications of Trees, Connectivity (15 hrs)

Applications of Trees – The connector problem.

Connectivity: Connectivity, Blocks and Applications, Construction of reliable communication Networks.

UNIT- IV : Euler Tours & Hamilton Cycles (15 hrs)

Euler tours, Euler Trail, Hamilton path, Hamilton cycles, Dodecahedron graph, Peterson graph, Hamiltonian graph, Closure of a graph.

UNIT- V: Applications of Eulerian Graphs (15 hrs)

Applications of Eulerian graphs, The Chinese postman problem, Fleury's Algorithm – The travelling salesman problem.

PRESCRIBED BOOK: Graph Theory with Applications by J.A. Bondy and U .S.R. Murthy, published by Mac. Millan Press.

REFERENCE BOOKS :

1. Graph theory with Applications by J.A.Bondy and U.S.R. Murthy published by Mac.Millan Press.
2. Introduction to Graph theory by S.Arumugham and S.Ramachandran, published by Scitech Publications, Chennai-17.
3. Graph Theory and Combinations by H.S.Govinda Rao published by Galgotia Publications.

BLUE PRINT FOR QUESTION PAPER PATTERN,

SEMESTER-VI

PAPER -VII, Elective VII D

UNIT	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Graphs and Subgraphs	01	01	01	14
II	Applications, Trees	01	01	01	14
III	Applications of trees, Connective	01	01	02	22
IV	Euler tours and Hamilton Cycles	01	01	02	22
V	Applications of Eulerian graphs	01	0	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1mark)
S.A.Q.= Short answer questions (5 marks)
E.Q.= Essay questions (8 marks)

Very Short answer questions : $5 \times 1 M = 05$
Short answer questions : $3 \times 5 M = 15$
Essay questions : $5 \times 8 M = 40$
Total Marks : = 60

P.R. Government College (Autonomous), Kakinada
III Year B.Sc. Degree Examinations VI Semester MATHEMATICS
Course (Elective VII (D) GRAPH THEORY
PAPER - VII : MODEL PAPER (w.e.f. 2019-20)

Time: 2 Hours 30 Min

Max. marks: 60 M

PART - I

Answer ALL the following questions. Each question carries 1 mark.

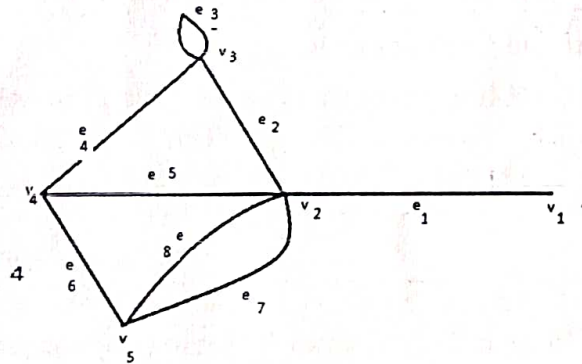
$5 \times 1 = 5 \text{ M}$

1. Define a simple graph.
2. How many edges have a tree with n vertices.
3. Define edge connectivity.
4. If a graph has an Euler trail then how many vertices have odd degree.
5. For tracing which type of graph Fleury's algorithm.

PART - II

Answer any THREE of the following questions. Each question carries 5 marks. $3 \times 5 = 15 \text{ M}$

6. Write the vertex set, edge set and degree of every vertex of the graph.



7. Prove that in a tree, any two vertices are connected by a unique path.
8. Define the vertex cut and edge cut of a graph $G(V,E)$ and give examples.
9. Define Eulerian graph and give an example.
10. Explain the travelling sales man problem.

PART - III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks.

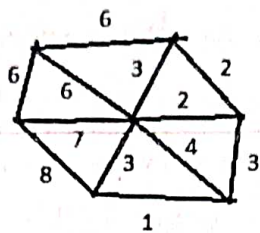
$5 \times 8 = 40 \text{ M}$

SECTION - A

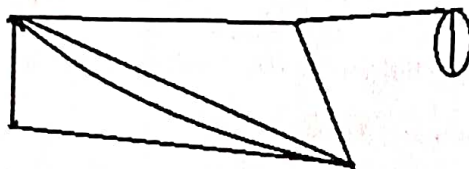
11. Define the degree of a vertex in a graph G and prove that the number of vertices of odd degree is even.
12. Write the Dijkstra's algorithm for finding the shortest path between two vertices in a graph and give an example.
13. Prove that a connected graph is a tree if and only if every edge is a cut edge.
14. If G is connected then prove that any two vertices of G lie on a common cycle.

SECTION - B

15. Write the Kruskal's Algorithm for finding a minimal spanning tree and find a minimal spanning tree of the following graph.



16. Draw the Dodecahedron graph and a Hamilton cycle of it.
17. Define blocks of a graph and draw the blocks of the following graph.



18. Write the Fleury's algorithm and give an example.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

III B.Sc. MATHEMATICS - VI Semester, (w.e.f. 2018-19)

Course (Cluster VIII A -1) INTEGRAL TRANSFORMS

Total Hours of Teaching-Learning : 75 @ 5 h/week

Total credits:05

Objectives:

- To be able to apply Laplace transform and inverse laplacetransform to find the solution of Ordinary Linear Differential Equations and Integral Equations.
- To understand the concepts of Infinite and Finite Fourier Transforms.
- To be able to find the Fourier transform of some functions.

UNIT – I : Application of Laplace Transform to solutions of Differential Equations (15 hrs)
Solutions of Differential Equations with Constants Co-efficient, Solutions of Differential Equations with Variable Co-efficient.

UNIT – II : Application of Laplace Transform (15 hrs)
Solution of Simultaneous Ordinary Differential Equations, Solutions of Partial Differential Equations.

UNIT – III : Application of Laplace Transforms to Integral Equations (15 hrs)
Definitions: Integral Equations - Abel's, Integral Equation - Integral Equation of Convolution Type, Integro Differential Equations - Application of L.T. to Integral Equations.

UNIT – IV : Fourier Transforms – I (15 hrs)
Definition of Fourier Transform – Fourier's Inverse Transform – Fourier Cosine Transform – Linear Property of Fourier Transform – Change of Scale Property for Fourier Transform – Sine Transform and Cosine Transform Shifting Property – Modulation Theorem.

UNIT – V : Fourier Transform - II (15 hrs)
Convolution Definition – Convolution Theorem for Fourier Transform – Parseval's Identity – Relationship Between Fourier and Laplace Transforms – Problems related to Integral Equations.

Finite Fourier Transforms :

Finite Fourier Sine Transform – Finite Fourier Cosine Transform – Inversion Formula for Sine and Cosine Transforms only statement and related problems.

Prescribed Text book:

Integral Transforms by A.R. Vasishtha and R.K. Gupta, Krishnaprakashan media Pvt. Ltd. Meerat.

Reference Books:

Integral Transforms by Dr. J. K. Goyal and K. P. Gupta, Pragati Prakashan.
M. D. Raisinghania Integral Transform, S. Chand & Co., New Delhi.

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-VI
PAPER VIII (A) 1, CLUSTER VIII (A) 1

UNIT	TOPIC	V.S. A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Application of Laplace Transform to solutions of Differential Equations	01	01	02	22
II	Application of Laplace Transforms	01	01	01	14
III	Application of Laplace Transforms to Integral Equations	01	01	01	14
IV	Fourier Transforms-I	01	01	02	22
V	Fourier Transform-II	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q. = Essay questions (8 marks)

Very Short answer questions : 5 x 1 M = 05
Short answer questions : 3 x 5 M = 15
Essay questions : 5 x 8 M = 40

Total Marks : = 60
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P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
III Year B.Sc. Degree Examinations VI Semester MATHEMATICS
Course (Cluster – VIII (A) -1) INTEGRAL TRANSFORMS
PAPER VIII (A) 1, MODEL PAPER (w.e.f. 2019-20)

Time: 2 hrs 30 Min

Max. Marks : 60M

PART – I

Answer **ALL** the following questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Write the formula of $L\{y''\}$.
2. Find $L\left(\frac{\partial y}{\partial x}\right)$.
3. Write the Integral equation of convolution type.
4. Write the Fourier Sine Transform of $F(x)$.
5. Find the cosine transform of $2e^{-5x}$.

PART – II

Answer any **THREE** of the following questions. Each question carries 5 marks. **3 x 5 = 15 M**

6. Solve $\frac{d^2y}{dx^2} + y = 0$ under the conditions that $y = 1, \frac{dy}{dx} = 0$ when $t = 0$.
7. Solve $(D^2 - 3)x - 4y = 0, x + (D^2 + 1)y = 0 \quad t > 0$
If $x = y = Dy = 0, Dx = 2$ when $t = 0$.
8. Solve the integral equation $F(t) = e^{-t} - 2 \int_0^t \cos(t - u) F(u) du$
9. Find the Fourier Transform of $F(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$.
10. Solve the integral equation $\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}$.

PART – III

Answer any **FIVE** questions from the following by choosing at least **TWO** from each section.
 Each question carries 8 marks. **5 X 8 = 40 M**

SECTION – A

11. Solve $(D + 1)^2 y = t$, give that $y = -3$, when $t = 0$ and $y = -1$, when $t = 1$.
12. Solve $\frac{\partial y}{\partial t} = 2 \frac{\partial^2 y}{\partial x^2}$, where $y(0, t) = 0 = y(5, t)$ and $y(x, 0) = 10 \sin 4\pi x$.
13. Solve the integral equation $\int_0^1 \frac{F(u) du}{(t-u)^{\frac{1}{3}}} = t(1+t)$.
14. Solve the integral equation $\int_0^t F(u) F(t-u) \, du = 16 \sin 4t$.

SECTION – B

15. Find the Fourier Cosine Transform of e^{-x^2} .
16. State and Prove Parseval's identity for Fourier Transforms.
17. Find the finite cosine transform of $f(x)$ if $f(x) = \frac{\cos k(\pi-x)}{k \sin k\pi}$.
18. Find the finite cosine transform of $(1 - \frac{x}{\pi})^2$.

P.R.GOVERNMENT COLLEGE (A), KAKINADA
III B.Sc. MATHEMATICS - VI Semester (w.e.f.2018-19)
Course (Cluster VIII (B)-1) Advanced Numerical Analysis

Total hours of teaching: 75 @ 5 hours/ week

Total credits:0 5

Objective:

- To find the integration and solutions for ordinary differential equations using numerical methods.
- To find the best fitted curve for the given data.

Unit – I : Curve Fitting (15 hrs)

Least – Squares Curve Fitting Procedures, Fitting a Straight Line, Nonlinear Curve Fitting, Curve Fitting by a Sum of Exponentials.

UNIT - II : Numerical Differentiation (15 hrs)

Derivatives using Newton's forward difference formula, Newton's backward difference formula, Derivatives using central difference formula, Stirling's interpolation formula, Newton's divided difference formula, Maximum and minimum values of a tabulated function.

UNIT- III : Numerical Integration (15 hrs)

General quadrature formula on errors, Trapezoidal rule, Simpson's 1/3 – rule, Simpson's 3/8 – rule, and Weddle's rules, Euler – Maclaurin Formula of summation and quadrature, The Euler transformation.

UNIT – IV : Solutions of Simultaneous Linear Systems of Equations (15 hrs)

Solution of linear systems – Direct methods: Matrix inversion method, Gaussian elimination methods, Gauss-Jordan Method, Method of factorization, Solution of Tridiagonal Systems, Iterative methods: Jacobi's method, Gauss-siedal method.

UNIT – V : Numerical solution of ordinary differential equations (15 hrs)

Introduction, Solution by Taylor's Series, Picard's method of successive approximations, Euler's method, Modified Euler's method, Runge – Kutta methods.

Reference Books :

1. Numerical Analysis by S.S. Sastry, Published by Prentice Hall India (Latest Edition).
2. Numerical Analysis by G. Sankar Rao, published by New Age International Publishers, New Hyderabad.
3. Finite Differences and Numerical Analysis by H.C. Saxena published by S.Chand and Company, Pvt. Ltd., New Delhi.
4. Numerical methods for scientific and engineering computation by M.K.Jain, S.R.K.Iyengar, R.K. Jain.

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-VI

PAPER VIII (B) 1, CLUSTER VIII (B) 1

UNIT	TOPIC	V.S.A.Q	S.A.Q(including choice)	E.Q(including choice)	Total Marks
I	Curve Fitting	01	01	01	14
II	Numerical Differentiation	01	01	01	14
III	Numerical Integration	01	01	02	22
IV	Solution of Linear System of Equations	01	01	02	22
V	Numerical Solutions for ODE	01	01	02	22
TOTAL		05	05	08	94

E.Q = Essay questions (8 marks)

S.A.Q = Short answer questions (5 marks)

V.S.A.Q = Very Short answer questions (1 mark)

Essay questions : $5 \times 1 M = 05$

Short answer questions : $3 \times 5 M = 15$

Very Short answer questions : $8 \times 5 M = 40$

Total Marks : _____ = 60

P.R. Govt. College (Autonomous), Kakinada
III B.Sc. Examination - VI Semester - Mathematics
(Cluster - VIII (B)-1) Advanced Numerical Analysis
PAPER-VIII (B) -1 MODEL PAPER (W.c.f. 2019-20)

Time: 2 hrs 30 Min

Max. Marks : 60M

PART - I

Answer ALL the following questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Write the normal equations for fitting a straight line.
2. Write the formula for $\frac{dy}{dx}$ at $x = x_1$.
3. Write Simpson's 3/8 formula.
4. Write the formula A^{-1} for a non singular matrix A.
5. Write Euler's formula for y_n

PART - II

Answer any THREE of the following questions. Each question carries 5 marks. 3 x 5 = 15 M

6. Find the least square line $y = a + bx$ for the data.

x_i	1	2	3	4	5
y_i	14	27	40	55	68

7. From the following table, find x correct to 4 decimal places for which y is minimum and find this value of y.

X	0.60	0.65	0.70	0.75
Y	0.6221	0.6155	0.6138	0.6170

8. Evaluate $\int_0^1 x^3 dx$ with five sub-intervals by Trapezoidal rule.

9. Solve the equation $x + y + z = 6$; $3x + 3y + 4z = 20$; $2x + y + 3z = 13$ using Gaussian elimination method.

10. Solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ using Picard's method upto 3 approximations.

PART - III

Answer any **FIVE** questions from the following by choosing at least **TWO** from each section.
Each question carries 8 marks. 5 X 8 = 40 M

SECTION - A

11. Fit a second degree polynomial to the following data by the method of least squares.

X	0	1	2	3	4
Y	1	1.8	1.3	2.5	6.3

12. Form the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for x = 1.5.

X	1.5	2.0	2.5	3.0	3.5	4.0
Y	3.375	7.0	13.625	24.0	38.875	59.0

13. Evaluate the $\int_0^{5.2} \log x dx$ using Weddle's Rule.

14. Derive Newton's general quadrature formula.

SECTION - B

15. Solve the equations $2x + 3y + z = 9$; $x + 2y + 3z = 6$; $3x + y + 2z = 8$ by factorization method.

16. Solve the following equations by Gauss-Seidel method

$$8x - 3y + 2z = 20; 4x + 11y - z = 33; 6x + 3y + 12z = 35;$$

17. Given $\frac{dy}{dx} = -xy^2$, $y(0) = 2$, compute $y(0.2)$ in steps of 0.1 using modified Euler's method.

18. Obtain the values of y at $x=0.1, 0.2$ using Runge-kutta method of fourth order for the differential equation $y' + y = 0$, $y(0) = 1$

PR GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank

PAPER – VIII (B)1: ADVANCED NUMERICAL ANALYSIS

Very Short Answer Questions

Unit-I

1. Define method of least squares.
2. Write the normal equations for fitting a straight line.
3. Find the normal equations for the curve $y = 1 - x^2$.
4. Find the normal equations for the curve $y = e^x$.
5. Find the normal equations for the curve $y = 1 + 4x - x^2$.

Unit-II

6. Write the Newton's back ward difference formula.
7. Write the first order difference formula which comes from Newton's forward Difference formula.
8. Write the first order difference formula by using stirling's formula.
9. Write the Newton's forward ward difference formula.
10. Write the second order difference formula which comes from Newton's backward Difference formula.
11. Write the second order difference formula which comes from stirling's formula.

Unit-III

12. Write the general formula for Numerical Integration.
13. Write the formula for finding the area by using the Trapezoidal rule.
14. Write the formula for finding the area by using Simpson's $\frac{1^{rd}}{3}$ - rule.
15. What is the value of n for which Simpson's $\frac{1^{rd}}{3}$ - rule can be applied.
16. What is the value of n for which Simpson's $\frac{3^{th}}{8}$ - rule can be applied.
17. Write the formula for finding the area by using Simpson's $\frac{3^{th}}{8}$ - rule.
18. Write the formula for finding the area by using Weddle's rule.

Unit-IV

19. The solution for the system of linear equations $AX = B$ exists by using the matrix inversion method is suitable, when _____.
20. Gauss elimination method reduces the system of equations to an equivalent _____ system.
21. In the method of factorization, the square matrix A can be written as $A = LU$, where $L = \underline{\hspace{2cm}}$.
22. In the method of factorization, the square matrix A can be written as $A = LU$, where $U = \underline{\hspace{2cm}}$.
23. The solution of tridiagonal system uses _____ method.
24. The number of iterations required to Gauss-Seidel method _____ than the number of iterations required to Gauss-Jacobi's method.

Unit-V

25. Write Taylor's series formula.
26. Write the iterative formula for Picard's method of successive approximations.
27. Write Euler's formula
28. Write Modified Euler's formula.
29. Write Rungu - Kutta formula for first order.
30. Write Rungu - Kutta formula for second order
31. Write Rungu - Kutta formula for third order

Short Answer Questions

Unit-I

1. Derive the normal equations of a straight line.
2. Find the least square line $y = a + bx$ for the data.

x_i	1	2	3	4	5
y_i	14	27	40	55	68

3. Derive the normal equations of a polynomial of second degree.
4. Fit a polynomial of the second degree to the data points given in the following table.

X	0.0	1.0	2.0
Y	1.0	6.0	17.0

5. Explain the procedure to fit a power function for the given data by the method of least squares

Unit-II

6. Derive the formulae for first and second order derivatives of y at $x = x_0$ by using the Newton's forward difference formula.
7. Derive the formulae for first and second order derivatives of y at $x = x_n$ by using the Newton's backward difference formula.
8. Derive the formulae for first and second order derivatives of y at $x = x_0$ by using the Stirling's interpolation formula.
9. From the following table of values of x and y , obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 0.4$.

X	0.0	0.1	0.2	0.3	0.4
Y	1.0000	0.9975	0.9900	0.9776	0.9604

10. From the following table of values of x and y , obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 3$.

X	1	2	3	4	5
Y	7.4036	7.7815	8.1291	8.4510	8.7506

Unit-III

11. Derive the Trapezoidal rule from general formula for numerical integration.
12. Find $I = \int_0^6 \frac{1}{1+x^2} dx$ by using Weddle's - rule with $h = 1$.
13. Derive Simpson's $\frac{3^{th}}{8}$ - rule from general formula for numerical integration.
14. Derive Simpson's $\frac{1^{rd}}{3}$ - rule from general formula for numerical integration.

15. Use the Euler-Maclaurin formula to prove $\sum_{x=1}^n x^2 = \frac{n(n+1)(2n+1)}{6}$

16. Derive Newton's general quadrature formula.

Unit-IV

17. Solve the system $2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$ by matrix inversion method.

18. Solve the equations $x + y + z = 6$, $3x + 3y + 4z = 20$, $2x + y + 3z = 13$ using Gauss elimination method.

19. Solve the system $5x - 2y + z = 4$, $7x + y - 5z = 8$, $3x + 7y + 4z = 10$ by using Gauss Jordan method.

20. Solve the system $5x - 2y + z = 4$, $7x + y - 5z = 8$, $3x + 7y + 4z = 10$ by using Gauss Seidal method.

Unit-V

21. Using Taylor's series, find $y(0.1)$ correct to four decimal places if $y' = x - y^2$ and $y(0) = 1$.

22. Solve the equation $y' = x + y^2$, subject to the condition $y = 1$ when $x = 0$ using Picard's method of successive approximations.

23. Solve the differential equation $y' = -y$ with the condition $y(0) = 1$ at $x=1$ by using Euler's method with $h = 0.1$.

Essay Questions

Unit-I

1. The table given below gives the temperature T (in $^{\circ}\text{C}$) and lengths l (in mm) of a heated rod. If $l = a_0 + a_1T$, find the best values for a_0 and a_1 .

T (in $^{\circ}\text{C}$)	20	30	40	50	60	70
l (in mm)	800.3	800.4	800.6	800.7	800.9	801.0

2. Determine the constants a and b by the method of least squares such that $y = ae^{bx}$ fits the following data.

X	2	4	6	8	10
Y	4.077	11.084	30.128	81.897	222.62

3. Determine the constants a and b by the method of least-squares such that $y = ae^{bx}$, fits the following data.

X	1.0	1.2	1.4	1.6
Y	40.170	73.196	133.372	243.02

4. Fit a function of the form $y = ax^b$ to the following data.

X	2	4	7	10	20	40	60	80
Y	43	25	18	13	8	5	3	2

5. Fit a function of the form $y = ax^b$ to the following data.

X	4	7	10	20	40
Y	25	18	13	8	5

Unit-II

6. From the following table of values of x and y , obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.2$.

X	1.0	1.2	1.4	1.6	1.8	2.0	2.2
Y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

7. From the following table of values of x and y , obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 3$.

X	0	1	2	3	4	5	6
Y	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309

8. From the following table of values of x and y , obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.6$.

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.9996

Unit-III

9. Find the area bounded by the curve and the X-axis from $x = 7.47$ to $x = 7.52$ for the following data by using Trapezoidal rule.

X	7.47	7.48	7.49	7.50	7.51	7.52
f(x)	1.93	1.95	1.98	2.01	2.03	2.06

10. Evaluate $I = \int_0^1 \frac{1}{1+x} dx$ correct to three decimal places by using Simpson's $\frac{1}{3}$ rd - rule with $h = 0.125$.

11. Estimate the value of the integral $\int_1^3 \frac{1}{x} dx$ by Simpson's $\frac{3}{8}$ th - rule.

12. Find the value of $\int_3^7 x^2 \cdot \log x dx$ by using weddle's rule with $h = 1$.

Unit-IV

13. By the method of factorization, solve $2x + 3y + z = 9$, $x + 2y + 3z = 6$, $3x + y + 2z = 8$.

14. Solve the system of equations $10x + 2y + z = 9$, $x + 10y - z = -22$, $-2x + 3y + 10z = 22$ by using Gauss-Jacobi's iteration method.

15. Solve the system of equations $10x + y - z = 11.19$, $x + 10y + z = 28.08$, $-x + y + 10z = 35.61$ by using Gauss-Seidel method.

Unit-V

16. Given differential equation $y'' - xy' - y = 0$ with the conditions $y(0) = 1$ and $y'(0) = 0$, determine the value of $y(0.1)$ by using Taylor's series method.

17. Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ with the initial conditions $y = 0$ when $x = 0$, use Picard's method to obtain y for $x = 0.25$.

18. Given $\frac{dy}{dx} = -xy^2$, $y(0) = 2$, compute $y(0.2)$ in steps of 0.1 using modified Euler's method.

19. Obtain the values of y at $x=0.1, 0.2$ using Runge-kutta method of fourth order for the differential equation $y' + y = 0$, $y(0) = 1$

P.R.GOVERNMENT COLLEGE (A), KAKINADA
B.Sc. THIRD YEAR MATHEMATICS - SEMESTER – VI (w.c.f. 2018-19)
Course : CLUSTER VIII- (C)-1 PRINCIPLES OF MECHANICS

No. Hours: 75 hrs

Credits : 05

Objectives:

- To understand the concepts of Mechanics which have applications in Physical problems.
- To Know the spherical co ordinate system.

Unit – I:

(15 hrs)

D'Alembert's Principle and Lagrange's Equations: Some definitions – Lagrange's equations for a Holonomic system – Lagrange's Equations of motion for conservative, nonholonomic system.

Unit – II:

(15 hrs)

Variational Principle and Lagrange's Equations: Variational Principle – Hamilton's Principle – Derivation of Hamilton's Principle from Lagrange's Equations – Derivation of Lagrange's Equations from Hamilton's Principle – Extension of Hamilton's Principle – Hamilton's Principle for Non-conservative, Non-holonomic system – Generalised Force in Dynamic System – Hamilton's Principle for Conservative, Non-holonomic system – Lagrange's Equations for Non-conservative, Holonomic system - Cyclic or Ignorable Coordinates.

Unit – III:

(15 hrs)

Conservation Theorem, Conservation of Linear Momentum in Lagrangian Formulation – Conservation of angular Momentum – conservation of Energy in Lagrangian formulation.

Unit – IV:

(15 hrs)

Hamilton's Equations of Motion: Derivation of Hamilton's Equations of motion – Routh's procedure – equations of motion – Derivation of Hamilton's equations from Hamilton's Principle – Principle of Least Action – Distinction between Hamilton's Principle and Principle of Least Action.

Unit – V:

(15 hrs)

Canonical Transformation: Canonical coordinates and canonical transformations – The necessary and sufficient condition for a transformation to be canonical – examples of canonical transformations – properties of canonical transformation – Lagrange's bracket is canonical invariant – poisson's bracket is canonical invariant - poisson's bracket is invariant under canonical transformation – Hamilton's Equations of motion in poisson's bracket – Jacobi's identity for poisson's brackets.

Reference Text Books :

1. Classical Mechanics by C.R.Mondal Published by Prentice Hall of India, New Delhi.
2. A Text Book of Fluid Dynamics by F. Charlton Published by CBS Publications, New Delhi.
3. Classical Mechanics by Herbert Goldstein, published by Narosa Publications, New Delhi.
4. Fluid Mechanics by T. Allen and I.L. Ditsworth Published by (McGraw Hill, 1972)
5. Fundamentals of Mechanics of fluids by I.G. Currie Published by (CRC, 2002)
6. Fluid Mechanics : An Introduction to the theory, by Chia-shun Yeh Published by (McGraw Hill, 1974).
7. Introduction to Fluid Mechanics by R.W Fox, A.T Mc Donald and P.J. Pritchard Published by (John Wiley and Sons Pvt. Ltd., 2003)

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
III B.Sc. MATHEMATICS - VI Semester (w.e.f. 2018-19)
Course (Elective VIII (D)-1) : APPLIED GRAPH THEORY

Total Hours of Teaching-Learning : 75 @ 5 h/week

Total credits:05

Objectives:

- To introduce the most application oriented field in Mathematics i.e., Graph Theory.
- To impart the awareness on applications of Graph Theory

Unit – I : Matchings (15 hrs)

Matchings – Alternating Path, Augmenting Path – Matchings and coverings in Bipartite graphs, Marriage Theorem, Minimum Coverings.

Unit – II : Perfect Matchings (15 hrs)

Perfect matchings, Tutte's Theorem, Applications, The Personal Assignment problem – The optimal assignment problem, Kuhn-Munkres Theorem

Unit – III : Edge Colorings (15 hrs)

Edge Chromatic Number, Edge Coloring in Bipartite Graphs – Vizing's theorem.

Unit – IV : Applications of Matchings, Independent sets and Cliques (15 hrs)

Applications of Matchings, The timetable problem, Independent sets, Covering number, Edge Independence Number, Edge Covering Number - Ramsey's theorem.

Unit – V : Ramsey's Number (15 hrs)

Determination of Ramsey's Numbers – Erdos Theorem, Turan's Theorem and Applications, Sehur's theorem, A geometry problem.

PRESCRIBED BOOK: Graph Theory with Applications by J.A.Bondy and U .S.R. Murthy published by Mac. Millan Press.

REFERENCE BOOKS :

1. Graph theory with Applications by J.A.Bondy and U.S.R.Murthy published by Mac.Millan Press
2. Introduction to Graph theory by S.Arumugham and S.Ramachandran, published by Scitech Publications, Chennai-17
3. Graph theory and combinations by H.S.Govinda Rao published by Galgotia Publications

BLUE PRINT FOR QUESTION PAPER PATTERN,

SEMESTER-VI

PAPER -VIII (D)-1, Cluster VIII D - 1

UNIT	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Matchings	01	01	02	22
II	Perfect matchings	01	01	01	14
III	Edge Colorings	01	01	01	14
IV	Applications of Matchings, Independent sets and Cliques	01	01	02	22
V	Ramsey's Number	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q. = Essay questions (8 marks)

Very Short answer questions : $5 \times 1 M = 05$
Short answer questions : $3 \times 5 M = 15$
Essay questions : $5 \times 8 M = 40$
Total Marks : = 60

P.R. Government College (A), Kakinada
III year B.Sc. Examination - Semester VI - Mathematics
Cluster - VIII (D) 1: Applied Graph Theory
PAPER VIII (D) 1 (MODEL PAPER w.e.f. 2019-20)

Time: 2 hours 30 Min

Maximum Marks: 60

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PART - I

Answer ALL the following questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Define bipartite graph.
2. Define Perfect Matching.
3. State Vizing's theorem.
4. Define an independent set.
5. State the Erdos Theorem.

PART - II

Answer any THREE of the following questions. Each question carries 5 marks. 3 x 5 = 15M

6. Define and give example of maximum and Perfect matching in graphs.
7. Prove that every 3-regular graph with out cut edges has a perfect matching.
8. Define an M-alternating tree and give an example.
9. Explain the time table problem with an example.
10. Give an example of (3,5) - Ramsey graph.

PART - III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks.

5 X 8 = 40 M

SECTION - A

11. In a bipartite graph, prove that the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.
12. Define and give example of M-Alternating Path and Covering of a graph.
13. Write the Kuhn - Munkers Algorithm.
14. If G is bipartite, then prove that $X' = \Delta$.

SECTION - B

15. A set $S \subseteq V$ is an independent set of G if and only if V/S is a covering of G
16. If $\delta > 0$, then prove that $\alpha' + \beta' = V$.
17. Prove that $r(k, l) \leq \binom{k+l-2}{k-1}$.
18. If a simple graph G contains no K_{m-1} , then prove that G is degree majorised by some complete m -partite graph H .

P.R.GOVERNMENT COLLEGE (A), KAKINADA
B.Sc. THIRD YEAR MATHEMATICS - SEMESTER – VI (w.e.f. 2017-18)

Course : CLUSTER VIII- (A,B)-2 SPECIAL FUNCTIONS

No. Hours: 75 hrs

Credits: 05

Objectives:

- To understand the concepts of special functions which have applications in Physical Sciences.
- To learn finding power series solutions to some special types of differential equations.

UNIT - I : HERMITE POLYNOMIAL (15 hrs)

Hermite Differential Equations, Solution of Hermite Equation, Hermite's Polynomials, Generating function, Other forms for Hermite Polynomial, To find first few Hermite Polynomials, Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials. (CHAPTER: 6.1 to 6.8)

UNIT - II : LAGUERRE POLYNOMIALS (15 hrs)

Laguerre's Differential equation, Solution of Laguerre's equation, Laguerre Polynomials, Generating function, Other forms for the Laguerre Polynomials, To find first few Laguerre Polynomials, Orthogonal property of the Laguerre Polynomials, Recurrence formula for Laguerre Polynomials, Associated Laguerre Equation. (CHAPTER: 7.1 to 7.9)

UNIT – III : LEGENDRE'S EQUATION (15 hrs)

Definition, Solution of Legendre's Equation, Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation(derivationis not required), To show that $P_n(x)$ is the coefficient of h^n in the expansion of $(1 - 2xh + h^2)^{1/2}$, Orthogonal properties of Legendre's Equation, Recurrence formulae, Rodrigue's formula. (CHAPTER: 2.1 to 2.8, 2.12)

UNIT - IV : BESSEL'S EQUATION (15 hrs)

Definition, Solution of Bessel's General Differential Equations, General solution of Bessel's Equation, Integration of Bessel's equation in series for $n=0$, Definition of $J_n(x)$, Recurrence formulae for $J_n(x)$, Generating function for $J_n(x)$. (CHAPTER: 5.1 to 5.7)

UNIT - V : BETA AND GAMMA FUNCTIONS**(15 hrs)**

Euler's Integrals - Beta and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions, Another form of Beta Function, Relation between Beta and Gamma Functions, Other Transformations. (CHAPTER: 2.9 to 2.15)

Prescribed text book: Special Functions by J.N. Sharma and Dr. R.K. Gupta.

BLUE PRINT FOR QUESTION PAPER PATTERN,**SEMESTER-VI, CLUSTER VIII -A, B, D-2****SPECIAL FUNCTIONS**

UNIT	TOPIC	V.S.A.Q 1 M	S.A.Q (including choice) 5 M	E.Q (including choice) 8 M	Marks Allotted
I	Hermit Polynomial	01	01	01	14
II	Laguerre Polynomial	01	01	01	14
III	Legendre's Equation	01	01	02	22
IV	Bessel's Equation	01	01	02	22
V	Beta And Gama Functions	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)

S.A.Q. = Short answer questions (5 marks)

E.Q. = Essay questions (8 marks)

Very Short answer questions : 5 x 1 M = 05

Short answer questions : 3 x 5 M = 15

Essay questions : 5 x 8 M = 40

Total Marks : _____ = 60

P.R. Government College (A), Kakinada
III B.Sc. Degree Examinations: Semester-VI, Mathematics
COURSE (Cluster VIII (A,B,D) 2) Special Functions
PAPER-VIII A,B, D 2 (MODEL PAPER w.c.f. 2019-2020)

Time: 2 hrs 30 Min

Max. Marks : 60 M

PART - I

Answer ALL the following questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Write the generating function of Hermit's polynomial.
2. Show that $L_1(x) = 1 - x$.
3. Define Legendre's equation.
4. Write $J_0(x)$.
5. Show that $\Gamma(1) = 1$.

PART - II

Answer any THREE of the following questions. Each question carries 5 marks.

3 x 5 = 15M

6. Evaluate $\int_{-\infty}^{\infty} x e^{-x^2} H_n(x) \cdot H_m(x) dx$.
7. Show that $L_2(x) = \frac{1}{2!}(2 - 4x + x^2)$.
8. Prove that $P_3(x) = \frac{1}{2}(5x^3 - 3x)$.
9. Prove that $J_{-n}(x) = (-1)^n J_n(x)$.
10. Evaluate $\int_0^a x^4 \sqrt{a^2 - x^2} dx$.

PART - III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks.

5 X 8 = 40 M

SECTION - A

11. State and Prove Rodrigue's formula for $H_n(x)$.
12. Prove that $xL_n''(x) + (1-x)L_n'(x) + nL_n(x) = 0$.
13. Prove that $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$.
14. Show that $\int_{-1}^1 P_m(x) \cdot P_n(x) dx = 0$ if $m \neq n$.

SECTION - B

15. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.
16. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
17. When n is a positive integer, prove that $\Gamma\left(-n + \frac{1}{2}\right) = \frac{(-1)^n 2^n \sqrt{\pi}}{1.3.5 \dots (2n-1)}$
18. Prove that $B(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$

PR GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
QUESTION BANK
PAPER-VIII B-2: SPECIAL FUNCTIONS

Very Short Answer Questions

Unit - I

1. Write the Hermite's Differential Equation.
2. Write the Hermite's Polynomial.
3. Write the generating function of Hermit's polynomial.
4. Write the Rodrigue's formula for $H_n(x)$.
5. Find the value of $H_0(x)$.
6. Write the orthogonal properties of Hermite's polynomial.

Unit -II

7. Define Laguerre's differential equation.
8. Write the Laguerre's polynomial.
9. Write the generating function of Laguerre's polynomial.
10. Write the Rodrigue's formula for Laguerre's polynomial.
11. Show that $L_1(x) = 1 - x$.
12. Write the orthogonal properties of $L_n(x)$.

Unit - III

13. Define Legendre's equation.
14. Write the Legendre's polynomial.
15. Write the general solution of Legendre's equation.
16. Write the generating function of Legendre's polynomial.
17. Write the Laplace's first integral for $P_n(x)$.
18. Write the Laplace's second integral for $P_n(x)$.
19. Write the orthogonal properties of $P_n(x)$.
20. Write the Rodrigue's formula for $P_n(x)$.
21. Show that $P_n(1) = 1$.

Unit - IV

22. Define Bessel's equation.
23. Write the Bessel's function of the first kind of order n.
24. Write the general solution of Bessel's equation.
25. Write $J_0(x)$.
26. When n is a positive integer, show that $J_{-n}(x) = (-1)^n J_n(x)$.

Unit - V

27. Define Gamma function.
28. Show that $\Gamma(1) = 1$.
29. Find $\Gamma\left(\frac{3}{2}\right)$.
30. Evaluate $\int_0^{\infty} x^4 \cdot e^{-x} dx$.

31. Define Beta function.
32. Write the relation between Beta and Gamma functions.
33. Write the Legendre's duplication formula.

Short Answer questions
Unit - I

1. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ and $H_{2n+1}(0) = 0$.
2. Find Hermite Polynomials for $n=0, 1, 2$.
3. Prove that $H_n'' = 4n(n-1)H_{n-2}$
4. Prove that $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$
5. Prove that, if $m < n$, $\frac{d^m}{dx^m} \{H_n(x)\} = \frac{2^m n!}{(n-m)!} H_{n-m}(x)$.
6. Evaluate $\int_{-\infty}^{\infty} x e^{-x^2} H_n(x) \cdot H_m(x) dx$.

Unit - II

7. Show that $L_n(x) = \frac{e^x}{n!} \frac{d^n (x^n e^{-x})}{dx^n}$.
8. Prove that $L_n'(x) = -\sum_{r=0}^{n-1} L_r(x)$.
9. Show that $L_2(x) = \frac{1}{2!} (2 - 4x + x^2)$.

Unit - III

10. Show that $P_n(x)$ is the coefficient of h^n in the expansion in ascending powers of $(1 - 2xh + h^2)^{-1/2}$.
11. Show that $P_n(1) = 1$ and $P_n(-x) = (-1)^n P_n(x)$.
12. Prove that $(2n+1)P_n = P_{n+1}' - P_{n-1}'$.
13. Prove that $(n+1)P_n = P_{n+1}' - xP_n'$.
14. Prove that $(1-x^2)P_n' = n(P_n - xP_n')$.
15. Prove that $P_3(x) = \frac{1}{2} (5x^3 - 3x)$.

Unit - IV

16. Prove that, when n is a positive integer $J_{-n}(x) = (-1)^n J_n(x)$.
17. Prove that $\frac{d}{dx} [x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$
18. Show that $J_n(-x) = (-1)^n J_n(x)$ for positive or negative integers.

Unit - V

19. Prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$
20. Evaluate $\int_0^a x^4 \sqrt{a^2 - x^2} dx$.
21. Evaluate $\int_0^{\infty} \frac{x^8 (1-x^6)}{(1+x)^{24}} dx$.
22. Prove that $\Gamma(n) = \frac{1}{n} \int_0^{\infty} e^{-y} y^{1/n} dy$ and hence show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

23. Evaluate $\int_0^2 x(8 - x^3)^{1/3} dx$.

24. Evaluate $\int_0^2 \frac{x^2}{\sqrt{2-x}} dx$

25. Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}} dx$.

26. Evaluate $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta$.

Essay Questions

Unit - I

1. State and Prove generating function of the Hermit's polynomial.
2. State and Prove Rodrigues formula for $H_n(x)$.
3. Find Hermite Polynomials for $n=0, 1, 2, 3, 4$ and 5 .
4. State and Prove Orthogonal Properties of Hermite Polynomials.
5. Prove that $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.
6. Prove that $H'_n(x) = 2nH_{n-1}(x) \quad n \geq 1$.

Unit - II

7. Prove that $\frac{1}{1-t} e^{-tx}/(1-t) = \sum_{n=0}^\infty t^n L_n(x)$
8. Find $L_0(x), L_1(x), L_2(x), L_3(x)$.
9. Prove that $\int_0^\infty e^{-x} L_n(x) L_m(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n \end{cases}$
10. $xL'_n(x) = nL_n(x) - nL_{n-1}(x)$
11. Prove that $xL''_n(x) + (1-x)L'_n(x) + nL_n(x) = 0$ and hence deduce that $L'_n(0) = -n$.
12. Prove that $(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)$

Unit - III

13. When n is a positive integer, prove that $P_n(x) = \frac{1}{\pi} \int_0^\pi [x \pm \sqrt{x^2 - 1} \cos \varphi]^n d\varphi$.
14. When n is a positive integer, prove that $P_n(x) = \frac{1}{\pi} \int_0^\pi [x \pm \sqrt{x^2 - 1} \cos \varphi]^n \frac{d\varphi}{[x \pm \sqrt{x^2 - 1} \cos \varphi]^{n+1}}$
15. Prove that $\int_{-1}^1 [P_n(x)]^2 dx = \frac{2}{2n+1}$.
16. Prove that $\int_{-1}^1 P_m(x) \cdot P_n(x) dx = 0$ if $m \neq n$.
17. Prove that $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$
18. Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's Polynomials.
19. Prove that $P_n(x) = \frac{1}{n! 2^n} \cdot \frac{d^n}{dx^n} (x^2 - 1)^n$.

20. Prove that $\int_{-1}^1 (x^2 - 1)P_{n+1}P_n' dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$.
21. Prove that $\int_{-1}^1 x^2 P_{n+1}P_{n-1} dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$.

Unit - IV

22. Prove that when n is a positive integer, $J_n(x)$ is the coefficient of z^n in the expansion of $e^{\frac{x(z-\frac{1}{z})}{2}}$ in ascending and descending powers of z .
23. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.
24. Prove that $xJ_n'(x) = -nJ_n(x) + xJ_{n-1}(x)$.
25. Prove that $\frac{d}{dx}(xJ_nJ_{n+1}) = x(J_n^2 - J_{n+1}^2)$.
26. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
27. Show that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
28. Prove that $\sqrt{\frac{\pi x}{2}} J_{3/2}(x) = \frac{1}{x} \sin x - \cos x$.
29. Show that $\cos x = J_0 - 2J_2 + 2J_4 - \dots$ and $\sin x = 2J_1 - 2J_3 + 2J_5 - \dots$

Unit V

27. Show that $\Gamma\left(n + \frac{1}{2}\right) = \frac{1.3.5\dots(2n-1)\sqrt{\pi}}{2^n}$, when n is a positive integer.
28. When n is a positive integer, prove that $\Gamma\left(-n + \frac{1}{2}\right) = \frac{(-1)^n 2^n \sqrt{\pi}}{1.3.5\dots(2n-1)}$
29. Prove that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
30. Show that $\Gamma\left(\frac{3}{2} - x\right)\Gamma\left(\frac{3}{2} + x\right) = \left(\frac{1}{4} - x^2\right)\pi \sec \pi x$ provided $-1 < 2x < 1$.
31. Show that (i) $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \pi\sqrt{2}$ (ii) $\Gamma(x)\Gamma(-x) = -\frac{\pi}{x \sin \pi x}$
32. Prove that $\int_0^{\pi/2} \sin^{2l-1} \theta \cdot \cos^{2m-1} \theta d\theta = \frac{\Gamma(l)\Gamma(m)}{2\Gamma(l+m)}$.

P.R. GOVERNMENT COLLEGE (A), KAKINADA

B.Sc. THIRD YEAR MATHEMATICS - SEMESTER – VI (w.c.f. 2018-19)

Course : CLUSTER VIII- (C)-2 FLUID MECHANICS

No. Hours: 75 hrs

Credits : 05

Objectives:

- To understand the concepts of Fluid Mechanics which have applications in water flow, blood flow etc.
- To Know the recent advances in fluid dynamics.

Unit - I:

(15 hrs)

Kinematics of Fluids in Motion - Real fluids and Ideal fluids – Velocity of a Fluid at a point – Streamlines and pathlines – steady and Unsteady flows – the velocity potential – The Vorticity vector – Local and Particle Rates of Change – The equation of Continuity – Acceleration of a fluid – Conditions at a rigid boundary – General Analysis of fluid motion.

Unit – II:

(15 hrs)

Equations of motion of a fluid- Pressure at a point in fluid at rest – Pressure at a point in a moving fluid – Conditions at a boundary of two inviscid immiscible fluids – Euler's equations of motion – Bernoulli's equation – Worked examples.

Unit – III:

(15 hrs)

Discussion of the case of steady motion under conservative body forces - Some flows involving axial symmetry – Some special two-dimensional flows – Impulsive motion – Some further aspects of vortex motion.

Unit – IV:

(15 hrs)

Some Two – dimensional Flows, Meaning of two-dimensional flow – Use of Cylindrical polar coordinates – The stream function – The complex potential for two-dimensional, Irrotational, Incompressible flow – Uniform Stream – The Milne-Thomson Circle theorem – the theorem of Blasius.

Unit – V:

(15 hours)

Viscous flow, Stress components in a real fluid – Relations between Cartesian components of stress – Translational motion of fluid element – The rate of strain quadric and principal stresses – Some further properties of the rate of strain quadric – Stress analysis in fluid motion – Relations between stress and rate of strain – the coefficient of viscosity and laminar flow - The Navier- Stokes equations of motion of a viscous fluid.

Reference Text Books :

1. A Text Book of Fluid Dynamics by F. Charlton Published by CBS Publications, New Delhi.
2. Classical Mechanics by Herbert Goldstein, published by Narosa Publications, New Delhi.
3. Fluid Mechanics by T. Allen and I.L. Ditsworth published by (McGraw Hill, 1972)
4. Fundamentals of Mechanics of fluids by I.G. Currie published by (CRC, 2002)
5. Fluid Mechanics, An Introduction to the theory by Chia-shun Yeh published by (McGraw Hill, 1974)
6. Fluids Mechanics by F.M White published by (McGraw Hill, 2003)
7. Introduction to Fluid Mechanics by R.W Fox, A.T Mc Donald and P.J. Pritchard published by (John Jiley and Sons Pvt. Ltd., 2003).

P.R. GOVERNMENT COLLEGE (A), KAKINADA

B.Sc. THIRD YEAR MATHEMATICS - SEMESTER – VI (w.e.f. 2018-19)

Course : CLUSTER VIII- (D) 2 DISCRETE MATHEMATICS

No. Hours: 75 hrs

Credits : 05

Objectives:

- To understand the concepts of Discrete Mathematics.
- To know the applications of Discrete Mathematics in Computer Science.

Unit - I:

(15 hrs)

Normal Forms – Disjunctive - Conjunctive Principal Disjunctive, Principal Conjunctive Normal Forms – Ordering and Uniqueness of Normal Forms.

Unit – II:

(15 hrs)

The theory of Inference for the statement Calculus - Rules of inferences - Consistency of Premises - Automatic Theorem proving.

Unit – III:

(15 hrs)

The predicate calculus - Inference Theory of the Predicate Calculus.

Unit – IV:

(15 hrs)

Lattices as partially Ordered sets - Lattices as Algebraic Systems - Boolean Algebra - Boolean Functions- Minimization.

Unit – V:

(15 hrs)

Finite - State Machines - Basic Concepts of Graph Theory - Basic Definitions – Paths - Reachability and Connectedness - Matrix Representation of Graphs - Trees.

Reference Text Books:

1. Scope and Standard as in the book “ Discrete Mathematical Structures With Applications To Computer Science” by Tremblay, J.P. & Manohar, R - Published by McGraw-Hill International Edition -1987 Edition.
2. “Discrete Mathematics & Graph Theory” by Bhavanari Satyanarana & Kuncham Syam Prasad, PHI Publications, New Delhi, Second Edition, 2014.
3. “Mathematical Foundation of Computer Science” by Bhavanari Satyanarayana, TV Pradeep Kumar, SK. Mohiddin Shaw, BS Publications, Hyderabad.2016.

LIST OF EXAMINERS & PAPER SETTERS IN MATHEMATICS

S.No.	Name of the Lecturer	Address
1	Sri. M. Venkata Rao	Lecturer in Mathematics, Government Degree College, Nidadavolu
2	Dr. K. Venkata Rao	Lecturer in Mathematics, Government Degree College, Alamuru.
3	Dr. D. Chitti Babu	Lecturer in Mathematics, Government Degree College, Tadepalligudem.
4	Smt. Gayatri	Lecturer in Mathematics, Government College (A), Rajamahendravaram.
5	Dr. P. Subhashini	Lecturer in Mathematics, Government Degree College, Ramachandrapuram.
6	Dr. D. Sai Baba	Lecturer in Mathematics, Sri YN College (A), Narasapur.
7	Dr. A.S.K. Chandra Shekar	Lecturer in Mathematics, DNR College (A), Bheemavaram.
8	Smt. K. Parameswari	Lecturer in Mathematics, Government Degree College, Ganapavaram.
9	Dr. Ch. Srinivas	Lecturer in Mathematics, Government College (A), Rajamahendravaram
10	Sri. K. Chitti Babu	Lecturer in Mathematics, Government Degree College, Ramachandrapuram.
11	Capt. K. Rama Krishna	Lecturer in Mathematics, Ms A.V.N College, Visakhapatnam.
12	Dr. V.S. Patnayak	Lecturer in Mathematics, M.R College, Vizaianagaram
13	Sri. K. Kameswara Rao	Lecturer in Mathematics, Government Degree College, Nidadavolu.
14	Ms. Y. Padmaja	Lecturer in Mathematics, Government Degree College, Pithapuram

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

DEPARTMENT OF MATHEMATICS

WORK LOAD FOR THE YEAR 2019-20 (ODD SEMESTERS)

Name of the Subject : Mathematics

Total No. of Hours : 210 (actual)

No. of Permanent posts sanctioned : 05

No. of Permanent staff working : 02

No. of Contract faculty : 00

No. of Part – Time Faculty : 03

S. No	Name of the class	No. of Theory hours	No. of Practical Hours	No. of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1	I MPC TM	6				6	
2	I MPC EM	6				6	
3	I MPE	6				6	
4	I MCAc	6				6	
5	I MCPc	6				6	
6	I MSAs	6				6	
7	I MPCs	6				6	
8	I MECs	6				6	
9	I MSCs	6				6	
10	I MCCs	6				6	
11	II MPC TM	6				6	
12	II MPC EM	6				6	
13	I MPE	6				6	
14	I MCAc	6				6	
15	II MCPc	6				6	
16	II MSAs	6				6	
17	II MPCs	6				6	
18	II MECs	6				6	
19	II MSCs	6				6	
20	II MCCs	6				6	
21	III MPC TM	5+5				10	
22	III MPC EM	5+5				10	
23	III MPE	5+5				10	
24	III MCPc	5+5				10	
25	III MSAs	5+5				10	
26	III MPCs	5+5				10	
27	III MECs	5+5				10	
28	III MSCs	5+5				10	
29	III MCCs	5+5				10	
Total Work load for the subject Mathematics						210	

GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
WORK LOAD FOR THE YEAR 2019-20 (ODD SEMESTERS)

Name of the Subject : Mathematics
 Total No. of Hours : 110 (adjusted)

No. of Permanent posts sanctioned : 05
 No. of Permanent staff working : 02
 No. of Contract faculty : 00
 No. of Part – Time Faculty : 03

S. No	Name of the class	No. of Theory hours	No. of Practical Hours	No. of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1	I MPC TM	6				6	
2	I MPC EM, MPE, MCAc	6				6	
3	I MCPc, MSAs	6				6	
4	I MPCs, MECs	6				6	
5	I MSCs, MCCs	6				6	
6	II MPC TM	6				6	
7	II MPC EM, MPE, MCAc	6				6	
8	II MCPc, MSAs	6				6	
9	II MPCs, MECs	6				6	
10	II MSCs, MCCs	6				6	
11	III MPC TM	5+5				10	
12	III MPC EM, MPE	5+5				10	
13	III MCPc, MSAs	5+5				10	
14	III MPCs, MECs	5+5				10	
15	III MSCs, MCCs	5+5				10	
Total Work load for the subject Mathematics						110	

In addition to these hours there are activity hours @ 2 hours for each class for 1st and 2nd years and 1 hour for 3rd year.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

DEPARTMENT OF MATHEMATICS

WORK LOAD FOR THE YEAR 2019-20 (EVEN SEMESTERS)

Name of the Subject : Mathematics
 Total No. of Hours : 204 (actual)
 No. of Permanent Postsanctioned : 05
 No. of Permanent staff working : 02
 No. of Contract faculty : 00
 No. of Part – Time Faculty : 03

S. No	Name of the class	No. of Theory hours	No. of Practical Hours	No. of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1	I MPC TM	6				6	
2	I MPC EM	6				6	
3	I MPE	6				6	
4	I MCAC	6				6	
5	I MCPc	6				6	
6	I MSAs	6				6	
7	I MPCs	6				6	
8	I MECs	6				6	
9	I MSCs	6				6	
10	I MCCs	6				6	
11	II MPC TM	6				6	
12	II MPC EM	6				6	
13	II MPE	6				6	
14	II MCAC	6				6	
15	II MCPc	6				6	
16	II MSAs	6				6	
17	II MPCs	6				6	
18	II MECs	6				6	
19	II MSCs	6				6	
20	II MCCs	6				6	
21	III MPC TM	5				5	
22	III MPC EM	5				5	
23	III MPE	5				5	
24	III MCPc	5				5	
25	III MSAs	5				5	
26	III MPCs	5				5	
27	III MECs	5				5	
28	III MSCs	5				5	
29	III MCCs	5				5	
16	Maths cluster	5+5				10	
17	Project work	5	-	-	-	5 for each project	
18	Analytical Skills	24	-	-	-	24	
Total Work load for the department of Mathematics						<u>204</u>	

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
WORK LOAD FOR THE YEAR 2019-20 (EVEN SEMESTERS)

Name of the Subject : Mathematics
 Total No. of Hours : 112 (adjusted)
 No. of Permanent Postssanctioned : 05
 No. of Permanent staff working : 02
 No. of Contract faculty : 00
 No. of Part – Time Faculty : 03

S. No	Name of the class	No. of Theory hours	No. of Practical Hours	No. of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1	I MPC (TM)	6				6	
2	I MPC EM, MPE, MCAC	6				6	
3	I MCPc, MSAs	6				6	
4	I MPCs, MECs	6				6	
5	I MSCs, MCCs	6				6	
6	II MPC (TM)	6				6	
7	II MPC EM, MPE, MCAC	6				6	
8	II MCPc, MSAs	6				6	
9	II MPCs, MECs	6				6	
10	II MSCs, MCCs	6				6	
11	III MPC (TM)	5				5	
12	III MPC EM, MPE	5				5	
13	III MCPc, MSAs	5				5	
14	III MPCs, MECs	5				5	
15	III MSCs, MCCs	5				5	
16	Maths cluster	5+5				10	
17	Project work	5	-	-	-	5 for each project	
18	Analytical Skills	24	-	-	-	12	
Total Work load for the department of Mathematics						112	

In addition to these hours there are activity hours @ 2 hours for each class for 1st and 2nd years and 1 hour for 3rd year.

P. R. GOVT. COLLEGE (A), KAKINADA
ACTION PLAN FOR THE ACADEMIC YEAR 2019-20

DEPARTMENT OF MATHEMATICS

S. No	Months	Week	Item as approved in BOS and to be incorporated in AC Meeting agenda as Institution Plan	Outcome of the activity
1	July, 2019	II	Mathematics Extension Lecture	Students Knowledge will be updated
2	December 2019	II	Town level Quiz and elocution computations	The competitive spirit will be improved among the students.
3	December 2019	II	Competitions to school children	The children may be inspired and put more concentration in Mathematics
4	December 2019	III	Celebration of Mathematics Day on 22 nd Dec -2019	The students will be motivated to pursue higher education in Mathematics.
5	February, 2020	IV	Science day celebrations	Students will get more interest to do projects and there is a scope to know the applicability of all subjects.
6	March, 2020	II	π Day celebrations	To impart the knowledge on the significance of π .